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ANALYSIS TECHNIQUES TO ASSESS AVAILABILITY  
IMPROVEMENT OF BUILT-IN-TESTS (BIT)

by

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  Systems of modules having individual BITs are studies for two failure-repair models: mutual independence of module states, or suspended animation (non-aging of good components during system downtime). Steady state availabilities are estimated using coherent structure techniques and empirically based estimates of BIT-related reduction in mean repair times.		

## TABLE OF CONTENTS

I.	INTRODUCTION .....	1
II.	DISCUSSION OF BIT USAGE .....	1
III.	STRUCTURE FUNCTION .....	5
	III.1     Situation where each subsystem is monitored by one BIT .....	9
IV.	SHORT OVERVIEW OF CLASSICAL RESULTS .....	11
	IV.1     ASSUMPTION: Processes in statistically independent blocks .....	11
	IV.2     ASSUMPTION: Suspended Animation .....	15
V.	AVAILABILITY CONSIDERATION .....	16
	V.1     BIT indications might send the system to main- tenance .....	16
	V.2     BIT indications ignored .....	20
	V.2.1.     Estimates for the mean time between failures .....	20
	V.2.2.     Estimates for the mean time to repair ..	23
	V.2.3.     Estimates for the system availability ..	30
VI.	STRUCTURES WHICH CAN BE REDUCED TO ONE SUB SYSTEM ONE BIT .....	47
	VI.1.     A central bit controller .....	47
	VI.2.     Blocks not equipped by BIT .....	48
	VI.3.     Subsystems monitored with several BIT's ..	49
	VI.4.     Several subsystems monitored by the same BIT .....	50
VII.	EXAMPLE .....	51
VIII.	CONCLUSIONS .....	61

## LIST OF FIGURES

1. Different scenarios - cases of BIT failures .....	3
2. Summary of assumptions .....	4
3. Definitions of Missed fault and type 1 error .....	6
4. Block declared "not OK" .....	7
5. Coherent structures .....	9
6. Availability viewpoint on the blocks .....	10
7. Relation between $\phi_R$ and $\phi_A$ .....	11
8. Mean time between failures as the function of design parameters .....	22
9. Mean time to repair in typical situation .....	27
10. No false alarms .....	27
11. Ideal BIT with no physical failures .....	28
12. BIT fast repair .....	29
13. Repair time improvement in cases 2,3 .....	30
14. Nomogram for % of improvement .....	34
15. Plot of marginal improvement .....	36
16. Partial derivatives of % improvement .....	40
17. Partial derivatives and sensitivity .....	41
18. Case 1, same as 15 .....	44
19. Case 1, same as 17 .....	45
20. Case 1, same as 18 .....	46
21. Centralized BIT network .....	48
22. Subsystem monitored by several BIT's .....	49

23. Several subsystems monitored by one BIT .....	50
24. Functional sketch of the radar .....	51
25. Reliability structure .....	51
26. Data .....	52
27. Organization into blocks .....	53
28. Area of improvement .....	54
29. Design .....	56
30. Data .....	57
31. Availability as a function of common FA-rate .....	58
32. Data case 1 .....	59
33. As 32 for case 1 .....	60

## SYMBOLS

A	-	availability	X	-	indicator function
$A_{av}$	-	long run average availability	x	-	percentage false
B	-	Expression	$x^*$	-	$x/(1-x)$
D	-	down-line	y	-	$u_i/u(s_j)$
d	-	differential	$\alpha$	-	percentage of reliability improvement
E	-	Expectation operator	$\Delta$	-	difference
F	-	distribution of failure times	$\xi$	-	$\lambda_{BITj}/\lambda_{sj}$
G	-	distribution of repair times	$\Lambda$	-	renewal function
g	-	function	$\lambda$	-	$d\Lambda/dt$
h	-	system function	$\Xi$	-	expected number of repairs
I	-	reliability importance	$\xi$	-	$d\Xi/dt$
L	-	index	n	-	$v(BITj)/v(s_i)$
ln	-	natural logarithm	v	-	MTTR
lim	-	limit	$\mu$	-	MTTB
MTTR	-	mean time to repair	$\Sigma$	-	sum
MTBF	-	mean time between failures	$\int$	-	integral
o	-	order (Landau symbol)	$\Phi$	-	structure
P	-	probability	$\psi$	-	substructure
t	-	time	[ ]	-	vector
U	-	uptime	[ ]*	-	augmented vector
u	-	$\xi \cdot \eta$	*	-	Stieltjes convolution

## Indexes

A	-	availability
R	-	reliability
i	-	j'th block
B <sub>j</sub>	-	j'th BIT indication
BIT <sub>j</sub>	-	j'th built in test of test equipment
s <sub>j</sub>	-	j'th subsystem
FA	-	false alarm
set up	-	set up time
FD/FI	-	failure detection/failure isolation
repl	-	replacement
ver	-	verification

## I. INTRODUCTION

Built in tests and built in test equipment, abbreviated BIT in this paper, are the names for hardware or software whose sole purpose in a system is to monitor its "health". By health we mean its operational status, readiness and availability. Supervising the system states a BIT should be also able to detect and isolate subsystem and component failures and so speed up repairs by minimally trained personnel.

Since costs of digital equipment are decreasing and subsystems are becoming more and more complicated, the need for BIT's will increase. This paper is an attempt towards studying the properties of BIT's in the context of reliability theory.

In particular, we examine the effect of BIT's on overall system availability. A perfect BIT will improve availability by reducing repair time. However, a BIT which itself fails frequently and which has a long repair time may have a net negative effect on the availability of the total system. Graphs are presented to quantitatively relate BIT failure and repair rates to system uptime and availability.

## II. DISCUSSION OF BIT USAGE

Consider a repairable system. A BIT should not under any circumstances, cause the system to fail. But a BIT is just another system, so it might also fail to perform its function properly. In such a case several possibilities might arise:

Case 1: A BIT "failure" might cause the system to stop its operation and to be sent to maintenance. For example: although a rocket navigational system shows apparently correct outputs, the supervising officer will not declare the system state as the state "Go" if a BIT indicates that something is wrong. Instead, a repair action will be requested.

Case 2: A Bit "failure" does not suspend the system operation, but the BIT is immediately repaired or replaced. This case is possible only when the BIT and the system are physically separated, as in acoustic monitoring of engines or turbines.

Case 3: A BIT "failure" does not cause the system functioning to stop, but its repair should wait for regular maintenance or for the system to breakdown. This is the most commonly encountered situation.

Case 4: A BIT "failure" is ignored. For example: most people don't care about the TV channel number indication, as long as they can see their favorite shows.

In the following discussion, we will be dealing with systems which consist of modules, subsystems or subassemblies. Whenever the system fails, the faulty unit is replaced by one in working condition. We will call such a unit a block.

We first discuss systems when each subsystem is monitored by exactly one BIT. Then we present systems which can be reduced to the above situation.

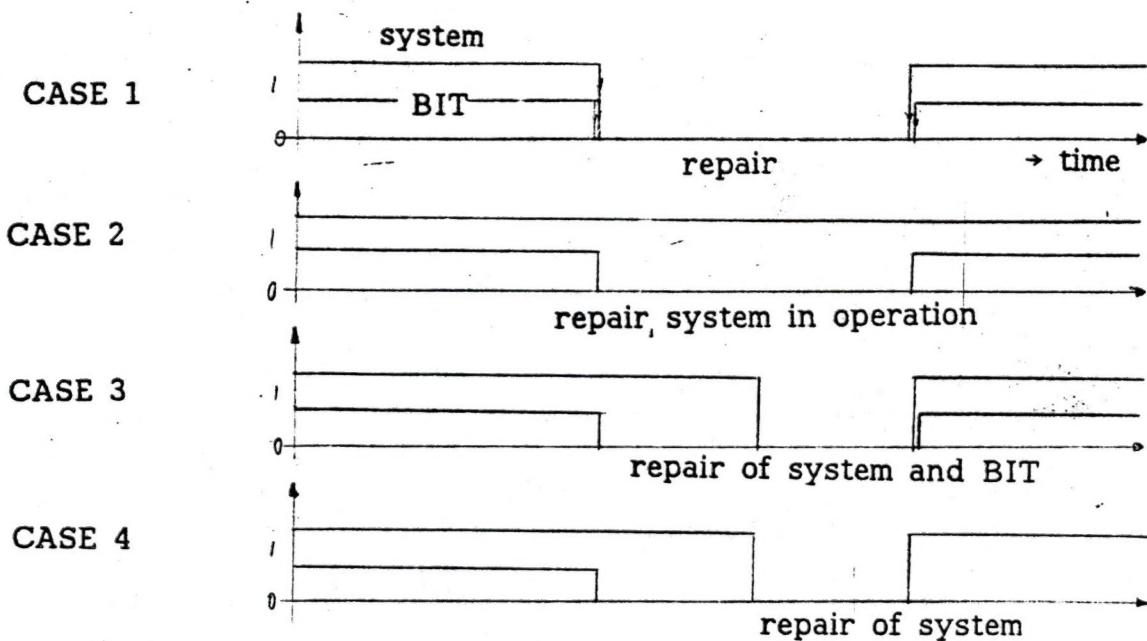


Figure 1: Different scenarios: cases of BIT failures in repairable systems.

Common cause failures such as breakdowns due to heat, vibrations, and radiation will cause the whole system to fail. Although these failures are unavoidable, we will not consider them here. Rather we will concentrate on system's failure which is caused by individual blocks failure. In this situation the assumption of independence between statistical properties of the system blocks can be justified. Mature designs and good protective measures against environment overstresses can keep the commoncause failures to a minimum.

The cases 1,2,3 and 4, described before, are treated under each of two different assumptions:

Assumption I: Failure-repair processes in different blocks are statistically independent. Blocks are separately maintained. Unrealistic in this assumption is that

blocks or subsystems are still in operation or at least aging, even when the system is down.

Assumption II: Blocks have a series-type reliability relation. When a block fails, the system fails and the other blocks don't function, so that these blocks cannot fail and do not age. We will refer to the situation as the state of a "suspended animation".  
(Barlow, 1982)

Although assumption I requires that there should be at least as many repair facilities as there are blocks in the system, this is not an essential constraint. We just don't want to introduce queuing problems into the consideration.

Real systems properties are somewhere between these two extreme assumptions, so their properties can be assessed by interpreting the results from these two cases.

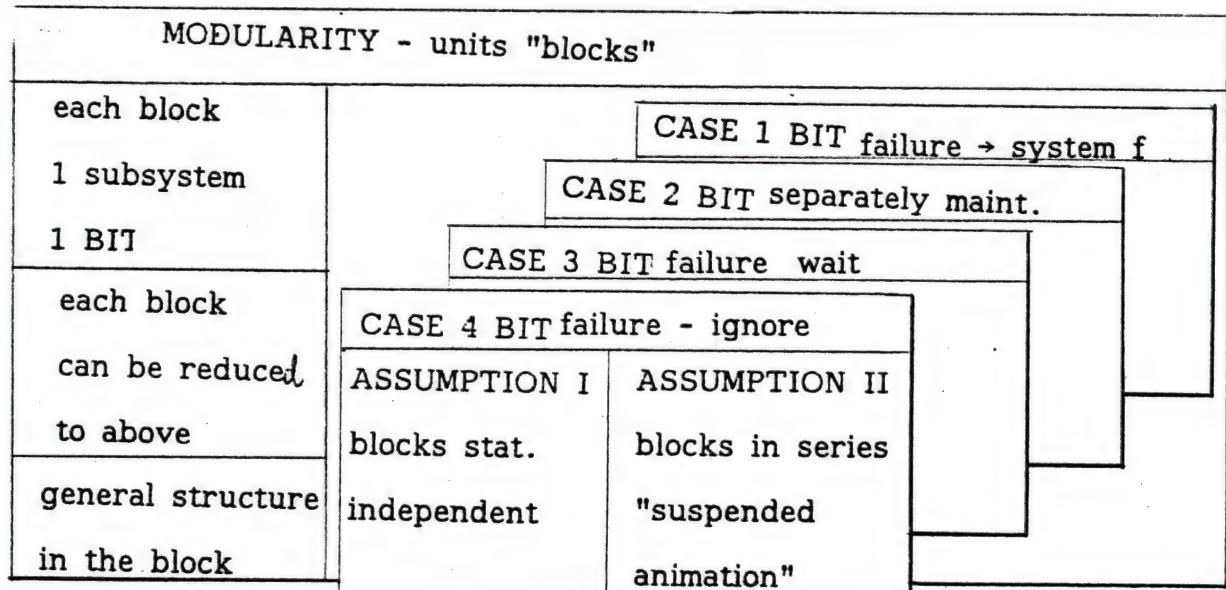


Figure 2: Summary of assumptions. Chapters are divided according to structure in the blocks. Each discussion is then subdivided into 4 cases each under 2 assumptions.

In the next chapter we first review the motions of structures and coherency and then we proceed to the availability consideration of BIT equipped systems.

### III. STRUCTURE FUNCTIONS

In this chapter the motions of indicator and structure functions are reviewed.

Let the indicator variable  $X_j(t)$  be defined:

$$X_j(t) = \begin{cases} 1 & j\text{'th block is in the operational state at time } t \\ 0 & j\text{'th block is "failed" at } t \end{cases} \quad (3.1)$$

Since the systems we are interested in contain components for performing the intended function and components for monitoring the state of "health", we introduce two more indicator functions. First  $X_{sj}(t)$  refers to the functional component  $j$ , which we will call the  $j$ th subsystem. Second  $X_{Bj}(t)$  refers to the BIT indication of the "health" state of  $j$ th subsystem.

$$X_{sj}(t) = \begin{cases} 1 & j\text{th subsystem is in the operational state at time } t \\ 0 & j\text{th subsystem is failed at } t \end{cases} \quad (3.2)$$

$$X_{Bj}(t) = \begin{cases} 1 & j\text{th BIT declares } j\text{th subsystem as "OK" at time } t \\ 0 & \text{BIT indications are "not OK" at } t \end{cases}$$

We refer for the "operational" state as "OK" state, rather than "not failed", because a unit does not necessarily have to fail or break down for the system to stop working. Unfortunately, it is in the nature of a BIT to occasionally show "wrong" status of the subsystem. Most BIT's determine the controlled system state by the measurement of parameters and use comparisons to some predetermined values. For example, a properly functioning system might show

voltages, temperatures, noise levels, vibration frequencies,... out of prescribed tolerance because of:

- normal system variability
- environmental variability
- noise
- interference
- graceful degradation
- transients
- tuning

All these influences can cause "outliers" with the result that "not go" or "not OK" indications appear.

BIT INDICATIONS		
SUBSYSTEM	"not OK"	"OK"
failed "not OK"	VALID	MISSED FAULT
not failed "OK"	Type 1 error	VALID

Figure 3: Definitions of Missed Fault and type 1 error.

Sometimes such "wrong" indications don't last long. So called "squawks" or intermittent failures will for example only bother the pilot, but if the maintenance personnel encounter them it is their duty to look up what is wrong. These are referred to as RTOK-Retest OK, CND - cannot Duplicate on BCS-Bench checks - serviceable "failures".

Since the "fail safe" design principle is usually applied to BIT implementations, missed faults do appear but are not in the magnitude of FA's and we will not elaborate on them. We do assume that true subsystem failures are self-evident. The BIT may give early warning

and help locate the fault within the subsystem.

But BIT can also physically fail or there might be some bugs in the BIT software.  $X_{BITj}(t)$  will then indicate such situations.

$$X_{BITj}(t) = \begin{cases} 1 & j\text{'th BIT physically operational at time } t \\ 0 & j\text{'th BIT physically failed at } t \end{cases} \quad (3.4)$$

We assume that a BIT failure always produces a "not OK" indication for the monitored block, meaning that no indication of failed BIT result in a maintenance action.

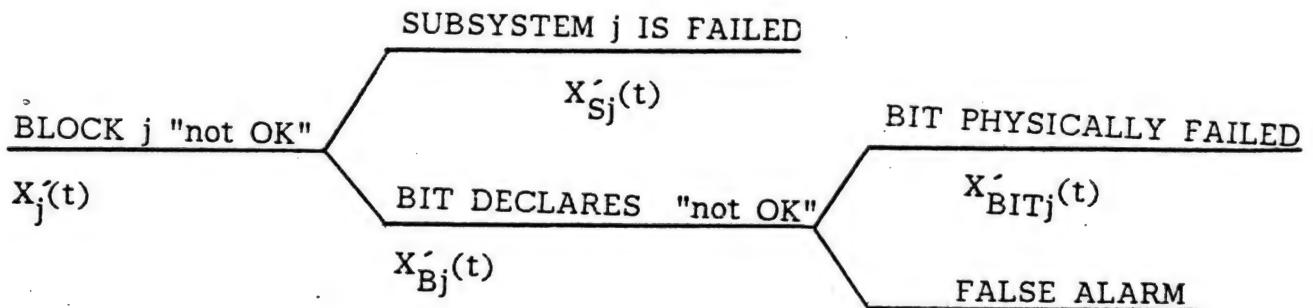


Figure 4: Block declared "not OK".  $X'(t)$  complement of  $X(t)$  indicator. Note that we define false alarm as an event when both BIT and subsystem are functioning, not taking in account BIT physical failures.

We will call a false alarm only the situation when the BIT shows "not OK" status but both the BIT and the corresponding subsystem are operational. These nomenclatures are not standard since some authors denote by the false alarm all the wrong BIT indications while we separated the BIT physical failures and software errors.

As the indicator variables were introduced on the components level, we will also introduce an indicator variable which will characterize the state of the whole system. Since a system consists of its elements, we will call the system's indicator variable a structure function of the system  $\phi$

$$\Phi([X_j(t)]) = \begin{cases} 1 & \text{the system is in the "OK" state at time } t \\ 0 & \text{otherwise at } t \end{cases}$$

where  $[X_j(t)] = (X_1(t), X_2(t), \dots, X_n(t))$  denotes a vector of the components indicators.

Most Reliability theory results relate to the coherent structures. The coherent structures are those in which every component affects or influences the system's state or more formally,

$$\Phi([X_j(t)]) \text{ is increasing in every } X_j(t) \text{ and every component is relevant.} \quad (3.6)$$

Addition of a BIT to a system should not affect its reliability, as mentioned in the beginning. So the structure of a BIT monitored system is not coherent. Obviously, we cannot afford to build an airplane which will crash just because the indication went wrong. So from the reliabilty viewpoint, the overall system including its primary function and BIT has a noncoherent structure. But when the system is maintained, the BIT plays a crucial role. Increased complexity will affect the systems reliability, but the faster repairs might still increase the availability of the system.

In classical systems most of the repair time is usually needed to locate a failed component. The BIT is here just to reduce this time. So the system is truly coherent from the availabilty and system's readiness viewpoint, although the structure is "noncoherent" with respect to reliability.

At this point, we introduce two structure functions  $\Phi_R$  and  $\Phi_A$ .  $\Phi_R$  the reliability or the classical structure describes the coherent structure or the relevant organization of subsystems to perform the intended function. On the other hand,  $\Phi_A$  availability structure, deals with time aspects of the system like readiness.

$$\Phi_R([X_j(t)]) = \begin{cases} 1 & \text{system is functioning "OK" at time } t \\ 0 & \text{system is "not OK" at } t \end{cases} \quad (3.7)$$

$$\Phi_A([X_j(t)]^*) = \begin{cases} 1 & \text{system is declared "OK" at } t \\ 0 & \text{system is not declared "OK" at } t \end{cases}$$

Where  $[X_j(t)] = (X_{s1}(t), X_{s2}(t), \dots, X_{sn}(t))$  is vector of the subsystems indicators and  $[X_j(t)]^* = (X_{s1}(t), X_{s2}(t), \dots, X_{sn}(t), X_{B1}(t), X_{B2}(t), \dots, X_{Bm}(t))$  is augmented vector of the BIT monitored system.

To discuss the relationship between the two structure functions, we first discuss the situation where every subsystem is monitored by only one BIT, and separate subsystems have separate BIT's.

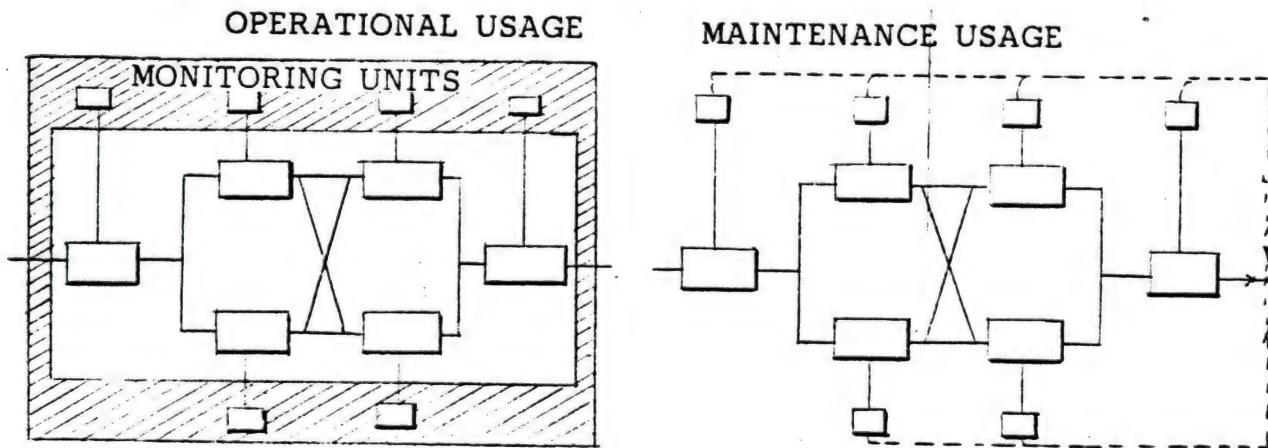


Figure 5: Coherent structure  $\Phi([X_j(t)])$ . During the system operation BITs should not influence the performance of the system (left). But when the system is maintained BIT should shorten repair times and thus increase availability and readiness.

### III.1 SITUATION WHERE EACH SUBSYSTEM IS MONITORED BY ONE BIT

Since every subsystem is monitored by exactly one BIT, we can bring the two together and call the new entity a block. Thus the jth block consists of the jth subsystem and the corresponding BIT.

A block is thus just an augmented subsystem.

When a system is maintained, a block will always be declared "OK" if both the subsystem and its BIT are functioning properly. When the subsystem is "not OK" and the BIT functions correctly, the block is not ready, it is "not OK". The same will happen when the subsystem is "OK" but the BIT indication is wrong.

SUBSYSTEM S <sub>j</sub>	BUILT-IN-TEST INDICATION B <sub>j</sub>	BLOCK j
OK	OK	OK
OK	not OK	not OK
not OK	OK	not OK
not OK	not OK	not OK

Figure 6: Availability viewpoint on the blocks.

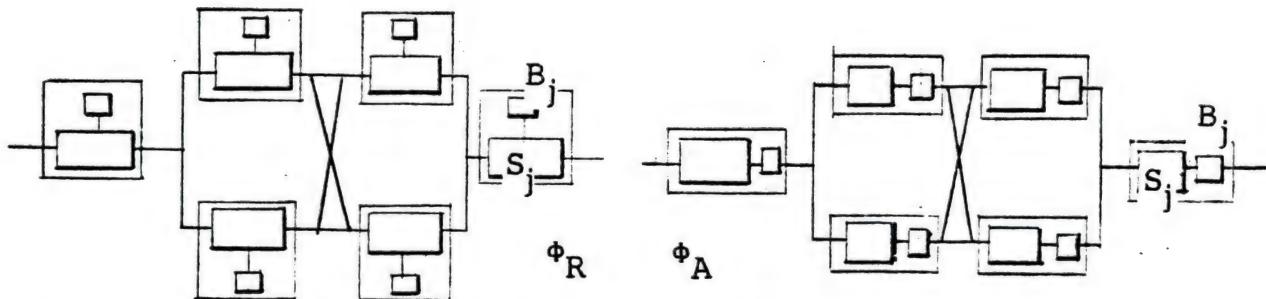
If  $X_j(t)$  is the indicator function of the blocks  $j$ ,  $X_{sj}(t)$  and  $X_{Bj}(t)$  are indicator variables of the  $j$ 'th subsystem and the corresponding BIT declaration then:

$$X_j(t) = X_{sj}(t) \cdot X_{Bj}(t) \quad (3.8)$$

So that for the availability purposes the subsystem and the corresponding BIT are connected in series:

$$\Phi_A([X_{sj}(t), X_{Bj}(t)]) = \Phi_R([X_{sj}(t) \cdot X_{Bj}(t)]) \quad (3.9)$$

where  $[X_{sj}(t), X_{Bj}(t)] = (X_{s1}(t), X_{B1}(t), X_{s2}(t), X_{B2}(t), \dots, X_{sn}(t), X_{Bn}(t))$  is the augmented vector of indicators of the subsystems and BIT's.  $[X_{sj}(t) \cdot X_{Bj}(t)] = (X_{s1}(t) \cdot X_{B1}(t), X_{s2}(t) \cdot X_{B2}(t), \dots, X_{sn}(t) \cdot X_{Bn}(t))$ , where the dot denotes the product.



**Figure 7:** Relation between  $\Phi_R$  and  $\Phi_A$  in situation where every subsystem  $S_j$  is monitored by separate BIT/BITE  $B_j$ . Noncoherent structure from reliability viewpoint becomes coherent with subsystem and BIT/BITE connected in series.

Since the addition of components always increases the complexity, we will now proceed to show the benefits and also the drawbacks of equipping systems with BIT's. To appreciate the addition of a BIT we have to look into the time behavior of the system's operation and repairs with some detail. We will limit ourselves to the patterns which can be described by alternating renewal processes.

#### IV. SHORT OVERVIEW OF CLASSICAL RESULTS

##### IV.1. ASSUMPTION I: PROCESSES IN STATISTICALLY INDEPENDENT BLOCKS

Let  $F_j$  be the distribution function of the failure times for the  $j$ th component and let  $G_j$  similarly be the p.d.f. for the repair times. The renewal function  $\Lambda_j(t)$  of the embedded renewal process of failures on  $(0,t)$  is by definition the expected number of failures of the  $j$ 'th component.

$$\Lambda_j(t) = \sum_{k=0}^{\infty} F_j^{(k+1)} * G_j^{(k)}(t) \quad (4.1)$$

where  $*$  denotes the Stieltjes convolution and  $(k)$  denotes  $k$ -fold re-

cursive convolution. For  $k=0$  let  $F^{(0)}$  be the unit step function.

Similarly, the embedded process of repairs can be described by

$$\Xi_i(t) = \sum_{k=0}^{\infty} F_j^{(k)} * G_j^{(k)}(t) \quad (4.2)$$

where  $\Xi_j(t)$  is the expected number of repairs on  $(0, t)$ .

$$\lambda_j(t) = \frac{d\Lambda_j(t)}{dt} \quad (4.3)$$

$$\xi_j(t) = \frac{d\Xi_j(t)}{dt} \quad (4.3)$$

$\lambda_j(t)$  and  $\xi_j(t)$  are corresponding renewal densities.

When there is no BIT present we can introduce the point availability  $A_j(t)$ :  $A_j(t) = \{X_j(t) = 1\}$ . It can also be expressed as:

$$A_j(t) = (1 - F_j(t)) + (1 - F_j(t)) * \Xi_j(t) \quad (4.4)$$

The component  $j$  is available if it is not failed or if it is repaired. We call  $A_j$  the time limit of  $A_j(t)$ , if it exists

$$A_j = \lim_{t \rightarrow \infty} A_j(t) = \frac{\mu_j}{\mu_j + \nu_j} \quad (4.5)$$

where the well established result includes  $\mu_j$  and  $\nu_j$  the means of  $F_j$  and  $G_j$  respectively.

For the coherent systems, the availability is the expected value of the system to be OK:

$$E[\Phi([X_j(t)])] = h([A_j(t)]) \quad (4.6)$$

where  $[X_j(t)] = (X_1(t), \dots, X_n(t))$ ,  $[A_j(t)] = (A_1(t), A_2(t), \dots, A_n(t))$  and  $h$  is called the (system) availability function. According to Barlow and Prochan (1975):

$$\lim_{t \rightarrow \infty} h([A_j(t)]) = h([A_j]) \quad (4.7)$$

Note also that if the system is not repairable

$$[\Phi(X_j(t))] = h([E(X_j(t))]) \quad (4.8)$$

Further: the probability that the system will fail in the interval  $(t, t+dt)$  denoted  $\lambda(t)dt$  is

$$\lambda(t)dt = \sum_{j=1}^n \lambda_j(t)I_j(t)dt + o(dt) \quad (4.9)$$

where we assumed that the probability of more than one failure in  $(t, t+dt)$  is of order  $dt-o(dt)$ .  $\lambda(t)$  and  $\lambda_j(t)$  are intensity functions of the system and the  $j$ th component failures.  $I_j(t)$  is the reliability importance (Birnbaum, 1967) of the  $j$ th component at time  $t$  and is defined:

$$I_j(t) = h(1_j, [A_i(t)]) - h(0_j, [A_i(t)]) \quad (4.10)$$

where  $(1_j, [A_i(t)]) = (A_1(t), \dots, A_{j-1}(t), 1, A_{j+1}(t), \dots, A_n(t))$  and  $(0_j, [A_i(t)])$  is similarly defined vector with zero on the  $j$ th spot.  $I_j(t)$  is the probability that the  $j$ 'th component will cause the system to fail at time  $t$ . The previous equation (4.9) shows that the intensities of the component failures should be weighted with their "criticalities" when we assess their influence on the system failure the importance function also has the properties:

$$I_j(t) = \frac{\partial h[A_j(t)]}{\partial A_j(t)} = \frac{\partial E(\Phi[X_j(t)])}{\partial E(X_j(t))} \quad (4.11)$$

$$I_j = \lim_{t \rightarrow \infty} I_j(t) = h(1_j, [A_i]) - h(0_j, [A_i]) \quad (4.12)$$

If  $\lambda(t)$  passes to a limit as  $t \rightarrow \infty$ :

$$\lim_{t \rightarrow \infty} \lambda(t) = \sum_{j=1}^n \frac{1}{\mu_j + v_j} I_j \quad (4.12)$$

Note that the renewal function  $\Lambda(t)$  is also where  $\sigma(t)$  is negligible and (Baxter 1983)

$$\lim_{t \rightarrow \infty} \frac{\Lambda(t)}{t} = \lim_{t \rightarrow \infty} \lambda(t) \quad (4.15)$$

Since  $I_j(t)$  is the probability that the failure of the  $j$ 'th component will cause the system's failure at time  $t$ , it is also the probability that the repair of the  $j$ 'th component will restore the system's function. So the probability that the system will be repaired in  $(t, t+dt)$ , denoted  $\xi(t)dt$ , is

$$\xi(t)dt = \sum_{j=1}^n \xi_j(t)I_j(t)dt + \sigma(dt) \quad (4.16)$$

and as above:

$$\Xi(t) = \int_0^t \xi(u)du = \sum_{j=1}^n \int_0^t \xi_j(u)I_j(u)du \quad (4.17)$$

where  $\Xi(t)$  is the expected number of repairs and as above:

$$\lim_{t \rightarrow \infty} \frac{\Xi(t)}{t} = \sum_{j=1}^n \frac{1}{\mu_j + v_j} I_j = \lim_{t \rightarrow \infty} \frac{\Lambda(t)}{t} \quad (4.18)$$

This shows that the expected number of repairs per unit time is equal to expected number of failures per unit time, after a long time.

Let  $U_1, U_2, \dots, U_k$  denote the successive uptimes, then

$$\lim_{k \rightarrow \infty} \frac{E[U_1+U_2+\dots+U_k]}{k} = \frac{h([A_j])}{\sum_{j=1}^n \frac{1}{\mu_j + v_j} I_j} \quad (4.19)$$

and similarly for  $D_1, D_2, \dots, D_k$  be the successive system downtimes:

$$\lim_{k \rightarrow \infty} \frac{E[D_1+D_2+\dots+D_k]}{k} = \frac{1-h([A_j])}{\sum_{j=1}^n \frac{1}{\mu_j + v_j} I_j} \quad (4.20)$$

Thus, the long run average system uptime and downtime are easily calculated.

All the above results are valid under the assumption I: Failure-repair processes in different component positions are assumed to be statistically independent. Next, results under the second assumption are reviewed.

#### IV.2 ASSUMPTION II: SUSPENDED ANIMATION

The following results hold for the series system in which the components are shut down until the failed component is fixed.

The long run average system availability  $A_{av}$  is defined as

$$A_{av} = \lim_{t \rightarrow \infty} \frac{1}{t} \int_0^t A(u)du = \frac{1}{1 + \sum_{i=1}^n \frac{\mu_i}{\mu_j}} \quad (4.21)$$

If the limit of  $A(t)$  exists, then it is equal to  $A_{av}$ . The limiting average expected number of the system failure caused by the  $j$ th component.

$$\lim_{t \rightarrow \infty} \frac{\Lambda_j(t)}{t} = \frac{A_{av}}{\mu_j} \quad (4.22)$$

$$\lim_{t \rightarrow \infty} \frac{\Lambda(t)}{t} = A_{av} \sum_{j=1}^n \frac{1}{\mu_j}$$

and long run average of the system uptimes (downtimes) is similarly as before

$$\lim_{k \rightarrow \infty} \frac{E(U_1 + U_2 + \dots + U_k)}{k} = \frac{1}{\sum_{j=1}^n \frac{1}{\mu_j} - N} = \mu \quad (4.23)$$

$$\lim_{k \rightarrow \infty} \frac{E(D_1 + D_2 + \dots + D_k)}{k} = \mu \sum_{j=1}^n \frac{\mu_j}{\mu_j} = \frac{1 - A_{av}}{A_{av}} \mu \quad (4.24)$$

## V. AVAILABILITY CONSIDERATIONS

The previous results will now be applied to BIT-monitored systems constructed of blocks which each contains one subsystem and its BIT. After a general discussion of failure rates in such systems, we develop approximate system availabilities by estimating the mean time to failure and the mean time to repair for a BIT-monitored block.

### V.1 FAILURE RATES OF BIT MONITORED SYSTEMS

#### Case 1: BIT indication might send the system to maintenance.

In this case, the BIT influences the system status. We've introduced two measures of the system effectiveness - namely the reliability and the availability so we defined two h-functions:

$$\begin{aligned} h_R([A_j(t)]) &= E[\Phi_R([X_j(t)])] \\ h_A([A_j(t)]^*) &= E[\Phi_A([X_j(t)]^*)] \end{aligned} \quad (5.1)$$

where  $[A_j(t)] = [E(X_{sj}(t))] = E([X_{sj}(t)])$  and  $[A_j(t)]^* = [A_{sj}(t), A_{Bj}(t)] = [E(X_{Bj}(t))]$ . As before, we discarded BIT components which are in  $h_R(\ )$ .

$$h_A([A_j^*(t)]) = E[\Phi_A([X_j(t)]^*)] = E(\Phi_A([X_{sj}(t), X_{Bj}(t)])) = E(\Phi_R([X_{sj}(t) \cdot X_{Bj}(t)]))$$

or

$$h_A([A_{sj}(t), A_{Bj}(t)]) = h_R([A_{sj}(t) \cdot A_{Bj}(t)]) \quad (5.2)$$

Since  $\Phi_A$  is coherent we can use the classical result for  $\lambda_A(t)dt$ , denoting the probability that the system will fail from the availability aspect:

$$\lambda_A(t)dt = \sum_{j=1}^n (I_{sj}(t)\lambda_{sj}(t) + I_{Bj}(t)\lambda_{Bj}(t)dt) + o(dt) \quad (5.3)$$

since in the discussed case 1 wrong BIT indications are also treated as "failures"  $\lambda_{sj}(t)$  is the failure rate of the j'th subsystem,  $\lambda_{Bj}(t)$  is the rate of "not OK" indication from the corresponding BIT.

$I_{sj}(t)$  and  $I_{Bj}(t)$  are the reliability importances associated with the subsystem and its BIT. These can be evaluated as

$$I_{sj}(t) = \frac{\partial h_A([A_j(t)]^*)}{\partial A_{sj}(t)} = \frac{\partial h_R([A_{sj}(t) \cdot A_{Bj}(t)])}{\partial A_{sj}(t)} = \\ \frac{\partial A_R([A_j(t)])}{\partial A_j(t)} = \frac{\partial A(t)}{\partial A_{sj}(t)}$$

$$I_{sj}(t) = I_j(t) A_{Bj}(t) \quad (5.4)$$

and similarly

$$I_{Bj}(t) = I_j(t) A_{sj}(t) \quad (5.5)$$

where  $I_j(t)$  is the reliability importance of the j'th block and where

$A_j(t) = A_{sj}(t) A_{Bj}(t)$  thus,

$$\lambda_A(t)dt = \sum_{j=1}^n I_j(t)[A_{Bj}(t)\lambda_{sj}(t) + A_{sj}(t)\lambda_{Bj}(t)]dt + o(dt) \quad (5.6)$$

From the above we establish two inequalities since  $A_{sj}(t), A_{Bj}(t) < 1$  it follows that

$$\lambda_A(t)dt < \sum_{j=1}^n I_j(t)[\lambda_{sj}(t) + \lambda_{Bj}(t)]dt = \sum_{j=1}^n I_j(t)\lambda_{sj}(t)dt + \\ \sum_{j=1}^n I_j(t)\lambda_{Bj}(t)dt \\ \lambda_A(t) < \lambda_S(t) + \lambda_B(t) \quad (5.7)$$

where  $\lambda_S(t)$  is the intensity of failures when the system consists of

blocks containing only subsystems, and  $\lambda_B(t)$  is similarly the intensity of failures when the system consists of blocks containing only the corresponding BIT.

$$\lambda_S(t)dt = \sum_{j=1}^n I_j(t)\lambda_{sj}(t)dt$$

$$\lambda_B(t)dt = \sum_{j=1}^n I_j(t)\lambda_{Bj}(t)dt$$

the rate with which the BIT equipped system will be sent to its maintenance is bounded above with the sum of 2 rates: first is the rate of the system consisting of only subsystems and no BIT's and second is the rate of the system where subsystems are replaced by its BIT's.

Also:

$$\lambda_A(t)dt > \sum_{j=1}^n I_j(t)\lambda_{sj}(t)A_{Bj}(t)dt \quad (5.8)$$

which follows from above (5.6) since all terms are positive. Since each BIT is designed to have high availability, the above inequality states not completely unexpected results: the BIT equipped systems have higher intensity of failures than the equivalent system without BIT if BIT can influence the decision about the system status (case 1).

When the system is in operation long enough so that it settles down to stationarity or steady state, we obtain:

$$\lambda_A = \lim_{t \rightarrow \infty} \lambda_A(t) = \sum_{j=1}^n I_j \left( \frac{1}{\mu_{sj} + v_j} A_{Bj} + \frac{1}{\mu_{Bj} + v_j} A_{sj} \right) \quad (5.9)$$

where  $\mu_{sj}$ ,  $\mu_{Bj}$  are mean times between failures of subsystem j and mean time between the BIT "no OK" indication.  $v_j$  is mean time to repair of the jth block (subsystem and BIT) for which an approximate

expression will be derived below.  $I_j$ ,  $A_{sj}$ ,  $A_{Bj}$  are limiting values of  $I_j(t)$ ,  $A_{sj}(t)$ ,  $A_{Bj}(t)$  respectively the inequalities 5.7 and 5.8 becomes:

$$A_B \lambda_S < \lambda_A < \lambda_S + \lambda_B \quad (5.10)$$

where  $A_B$  is a constant such that for all BIT's  $A_B < A_{Bj}$ , and  $\lambda_S$ ,  $\lambda_B$  are the steady state values of  $\lambda_S(t)$  and  $\lambda_B(t)$  respectively.

The formula holds for the assumption I of independent blocks.

For the suspended animation (assumption II) we get similar results. But since blocks are assumed to be connected in series reliability importance does not appear.

$$\lim_{t \rightarrow \infty} \lambda_A(t) = \frac{1}{1 + \sum_{j=1}^n \frac{v_j}{\mu_{sj}}} \sum_{j=1}^n \frac{1}{\mu_{sj}} + \frac{1}{1 + \sum_{j=1}^n \frac{v_j}{\mu_{Bj}}} \sum_{j=1}^n \frac{1}{\mu_{Bj}} \quad (5.11)$$

### Case 2, 3 and 4 Bit indications ignored

In contrast to the previous case, here the BIT declarations do not influence decisions of the system status, as long as the system is in the operation. Thus the failure rate stays the same as without BIT's

$$\lambda(t)dt = \sum_{j=1}^n I_j(t) \lambda_{sj}(t)dt + o(dt) \quad (5.12)$$

To evaluate availabilities we will use the asymptotic values, because the above assumptions guarantee their existance. To repeat:

$$A = h([A_j]) \quad (5.21)$$

$$A_j = \frac{\mu_j}{\mu_j + v_j} = \frac{1}{1 + \frac{v_j}{\mu_j}}$$

A...availability of the system  
 Aj...availability of the j'th block  
 μj...meantime between failures for the jth block.  
 vj...meantime between repairs for the jth block.

The above is valid under assumption I of independent blocks.

Under assumption II of the suspended animation in series:

$$A_{av} = \frac{1}{1 + \sum_{j=1}^n \frac{v_j}{\mu_j}} \quad (5.22)$$

### V2.1 ESTIMATES FOR THE MEANTIME BETWEEN FAILURES $\mu_j$

Obviously in the cases 2, 3 and 4 where BIT indications are ignored when the subsystem is in the operation, the mean time between failures  $\mu_j$  is just equal to the mean time between failures of the jth subsystem  $\mu_{sj}$ :

$$\text{Case 2, 3, 4} \quad \mu_j = \mu_{sj} \quad (5.23)$$

But in the case 1, the BIT "not OK" indications might send the system to the maintenance as described on Figure 4, page 5.

Three distinct events might send our system to repair by putting the j'th block to "not OK" state: either the j'th subsystem fails (I) or the j'th BIT fails (II) or the false alarms occurs (III):

$$\mu_j = \mu_{sj}P(1) + \mu_{BIT+j}P(2) + \mu_{FAj}P(3) \quad (5.24)$$

where P(1) is the probability that the subsystem fails before the occurrence of the BIT failure or false alarm, or that I happens first, P(2) is the probability that II occurs first and P(3) is the probability that the false alarm sends out the block to the state "not OK".

We will assume that the evaluation of the system availability is performed in its design phase. The availability we are interested in, is then of the system in its mature state—excluding infant mortality and wearout period. Furthermore without the loss of generality we will assume that our subsystems and BIT's are complex systems by themselves so that "quasi" constant failure rate might be applied to

them. The above assumptions are necessary since the only available data in the design phase are usually of the constant failure rates. Although such an estimate is often not completely justified in real cases it is quite good for comparison purpose in the design phase.

Constant failure rates yield a Poisson process for the occurrence of failures. The probability that i'th cause occurs first among n of the possible events is:

$$p(i) = \frac{\lambda_i}{\lambda} \quad (5.25)$$

where  $\lambda_i$  is the i'th failure rate and  $\lambda$  is the failure rate of the sum of the n independent processes which is also a Poisson process.

$$\begin{aligned} P(1) &= \frac{\lambda_{sj}}{\lambda_j} \\ P(2) &= \frac{\lambda_{BITj}}{\lambda_j} \\ P(3) &= x \end{aligned} \quad (5.26)$$

where  $\lambda_j$  is the failure rate of j'th blocks,  $\lambda_{sj}$  and  $\lambda_{BITj}$ , are failure rates of the subsystem j and of its BIT physical failures  $P(3) = x$  is the percentage rate of the false alarms, normalized on the j'th block failure rate. Note that x is not a false alarm rate, but rather the false alarm percentage of all events which occurs to the block. Since something has to cause the block failure:

$$P(1) + P(2) + P(3) = 1$$

or

$$\begin{aligned} \frac{\lambda_{sj} + \lambda_{BITj}}{\lambda_j} + x &= 1 \\ \lambda_j &= \frac{\lambda_{sj} + \lambda_{BITj}}{1-x} \end{aligned} \quad (5.27)$$

and since failure rates are assumed to be constant:

$$\mu_j = \frac{1}{\lambda_j} = \frac{1-x}{1 + \frac{\lambda_{BITj}}{\lambda_{sj}} \mu_{sj}} \quad (5.28)$$

or

Case 1

$$\frac{\mu_j}{\mu_{sj}} = \frac{1}{1 + \frac{\lambda_{BITj}}{\lambda_{sj}} (1-x)} \quad (5.29)$$

where  $\lambda_{sj}$  and  $\lambda_{BITj}$  are easily obtained or prescribed from the design and  $x$  is the percentage rate of false alarms. Equation (5.29) expresses the mean time between failures  $\lambda_j$ , for the systems with BIT in terms of parameters which are natural to estimate or prescribed as the system is being designed.

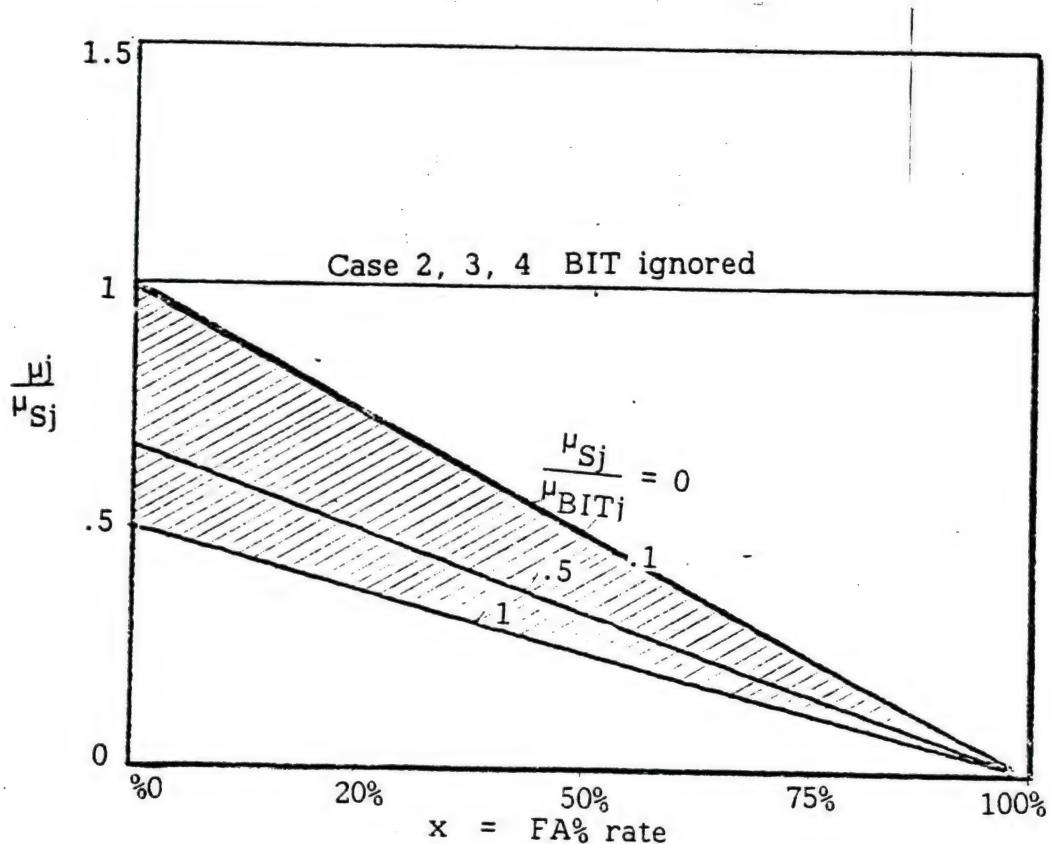


Figure 8: Mean time between failures as the function of design parameters and percentage false alarms  $x$ .

For example: If we can tolerate 5 to 10% of false alarms, and if BIT is constructed from 2 to 10 times better parts than the monitored subsystem, the new MTBF will be only 60 to 86% of original MTBF of the subsystem.

## V2.2 ESTIMATES FOR THE MEAN TIME TO REPAIR $v_j$

To estimate mean time between repairs we proceed as above, with one very important difference: we will not assume anything about the repair time distribution, as we did with failures.

In the case 1, the time to complete repairs is a function of what has caused the maintenance operation on the jth block:

- 1 - the subsystem has failed
- 2 - the BIT has physically failed
- 3 - false alarm

In the case 2, and 3, the block goes to repair only when the subsystem fails, but the time to repair depends on the situation:

- 1 - only the subsystem has failed
- 2 - the BIT has failed before (or has not been repaired yet)
- 3 - false alarms occurred before

So in cases 1, 2 and 3, if we denote by  $\lambda_{sj=0}$ ,  $v_{BITj=0}$ ,  $v_{FA}$ , mean time to repair for the 1st, 2nd or 3rd cause respectively then

$$\text{Case 1,2,3} \quad v_j = \lambda_{sj=0} P(1) + \lambda_{BITj=0} P(2) + v_{FA} P(3) \quad (5.30)$$

$$\text{Case 4} \quad v_j = v_{sj} \quad (5.31)$$

while in the case 4 repair of BIT is omitted. Note that  $P(1)$ ,  $P(2)$ , and  $P(3)$  in cases 1 and 3 are the probabilities that the j'th sub-

system, its BIT or that false alarm occurs first, before the other two respectively:

$$P(1) = \frac{\lambda_{sj}}{\lambda_j} = \frac{1-x}{\lambda_{sj} + \lambda_{BITj}} \lambda_{sj} = \frac{1-x}{1 + \frac{\lambda_{BITj}}{\lambda_{sj}}} \quad (5.35)$$

$$P(2) = \frac{\lambda_{BITj}}{\lambda_j} = \frac{\lambda_{sj}}{\lambda_j} \frac{\lambda_{BITj}}{\lambda_{sj}} = \frac{1-x}{1 + \frac{\lambda_{BITj}}{\lambda_{sj}}} \frac{\lambda_{BITj}}{\lambda_{sj}}$$

$$P(3) = x$$

But in the case 2, where BIT's are repaired separately  $\lambda_{BITj}$  should be substituted only with the rate of failures which are not repaired at the subsystem failures. We will omit details here, since all the other derivations do not change.

To estimate the mean time to repair  $v_j$  in the above equation (5.30) we divide the repair time into several stages:

set up for tests

failure detection, failure isolation (FD/FI)

replacement

verification

the time to repair the subsystem or to repair the BIT will be the sum of times needed to accomplish those separate tasks:

$$v(sj) = v_{set\ up}(sj) + v_{FD/FI}(sj) + v_{repl}(sj) + v_{ver}(sj) \quad (5.33)$$

$$v(BITj) = v_{set\ up}(BITj) + v_{FD/FI}(BITj) + v_{repl}(BITj) + v_{ver}(BITj)$$

where  $v(sj)$  and  $v(BITj)$  are mean repair times to the  $j$ 'th subsystem and the  $j$ 'th BIT repair and  $v_{set\ up}( )$ ,  $v_{FD/FI}$ ,  $v_{repl}( )$ ,  $v_{ver}( )$  correspond to the repair stages, described above.

The mean time to repair of the j'th block, when the subsystem failed but BIT functions properly -  $v_{sj=0}$  is:

$$v_{sj=0} = v(sj) - v_{FD/FI}(sj) \quad (5.34)$$

since failure detection and isolation is provided by the BIT and practically no time is spent in comparison with other tasks.

The time to repair is in every specific ease different and depends on factor such as:

training of the personnel

skill level of the personnel

available equipment

and others. In a typical case the expected time to repair will take 10% for set up, 50% for the trouble shooting, 30% for the replacement and the remaining 10% for the verification (      ). The validity of this assumption can be checked in the existing equipment and then transferred to the new designs.

$$v_{\text{set up}}(\cdot) = .1v(\cdot)$$

$$v_{FD/FI}(\cdot) = .5v(\cdot)$$

$$v_{\text{repl}}(\cdot) = .3v(\cdot) \quad (5.35)$$

$$v_{\text{ver}}(\cdot) = .1v(\cdot)$$

This assumption of nominal relations among durations of portions of the repair cycle is fundamental to the assessment of the contribution of BIT to system availability. The equation (5.34) might be now rewritten:

$$v_{sj=0} = 0.5v(sj) \quad (5.36)$$

When BIT physically fails, the complete maintenance of the corresponding block consists of discovering the condition and replacing the BIT. No part in the subsystem need to be replaced in the case 1,

but in cases 2, 3 the subsystem is always repaired, since it was the cause of maintenance.

$$\text{Case 1 } v_{\text{BITj}=0} = v(sj) - v_{\text{repl}}(sj) + v(\text{BITj}) = 0.7v(sj) + v(\text{BITj})$$

$$\text{Case 2,3 } v_{\text{BITj}=0} = v(sj) + v(\text{BITj}) \quad (5.37)$$

In this case of false alarm, nothing really fails in case 1, which in cases 2 and 3, subsystem should be again repaired:

$$v_{\text{FAj}} = v(sj) - (v_{\text{repl}}(sj)) + v(\text{BITj}) - v_{\text{repl}}(\text{BITj}) \quad (5.38)$$

$$\text{Case 1 } v_{\text{FAj}} = .7 v(sj) + .7v(\text{BITj})$$

$$\text{Case 2,3 } v_{\text{FAj}} = v(sj) + .7v(\text{BITj})$$

The above results are now plugged into the final equation:

$$\text{Case 1: } v_j = .5v(sj)P(1) + [.7v(sj)+v(\text{BITj})]P(2) + .7[v(sj)+v(\text{BITj})]P(3)$$

$$\text{Case 2,3: } v_j = .5v(sj)P(1) + [v(sj)+v(\text{BITj})]P(2) + [v(sj)+.7v(\text{BITj})]P(3)$$

After some manipulation

$$B_1 = ((.5 + \frac{\lambda_{\text{BITj}}}{\lambda_{sj}}) \frac{v(\text{BITj})}{v(sj)} + .7 \frac{\lambda_{\text{BITj}}}{\lambda_{sj}} \frac{1}{1 + \frac{\lambda_{\text{BITj}}}{\lambda_{sj}}})$$

$$\frac{v_j}{v(sj)} = B_1 + [.7 (1 + \frac{v(\text{BITj})}{v(sj)}) - B_1]x$$

(5.39)

Case 1

$$B_{2,3} = ((.5 + \frac{\lambda_{\text{BITj}}}{\lambda_{sj}}) \frac{v(\text{BITj})}{v(sj)} + \frac{\lambda_{\text{BITj}}}{\lambda_{sj}}) \frac{1}{1 + \frac{\lambda_{\text{BITj}}}{\lambda_{sj}}}$$

$$\frac{v_j}{v(sj)} = B_{2,3} + [1 + .7 \frac{v(\text{BITj})}{v(sj)} - B_{2,3}]x$$

(5.40)

Case 2,3

Although the above equation for the mean time to repair the j'th block might seem a little awkward, it is really quite simple. To appreciate its meaning, we draw some plots for case 1.

1. Typical plot;  $\frac{\lambda_{BITj}}{\lambda_{Sj}} = .1$
- 

$$y = .51 + .09\eta + [.19 + .91\eta]x \quad y = \frac{v_j}{v(Sj)} \quad \eta = \frac{v(BITj)}{v(Sj)} \quad (5.41)$$

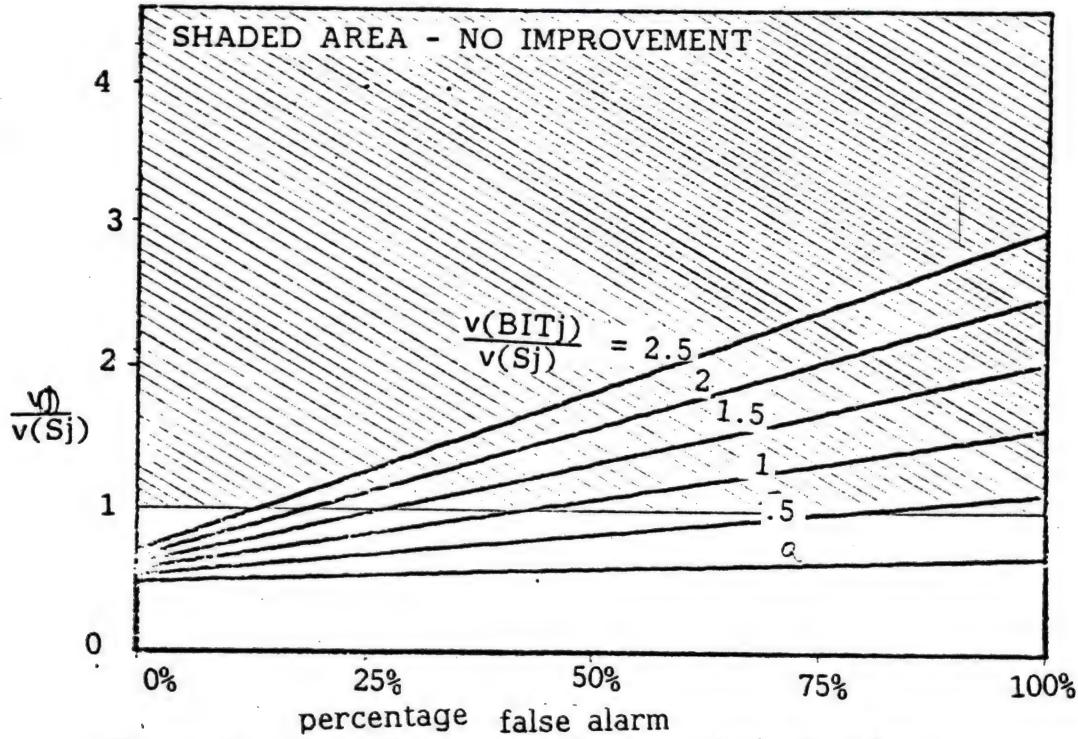


Figure 9: Mean time to repair in typical situation.

The shaded area represents the area in mean repair time due to inclusion of the BIT. The influence of the false alarms is clearly seen.

2. Ideal system, with no false alarms,  $x=0$

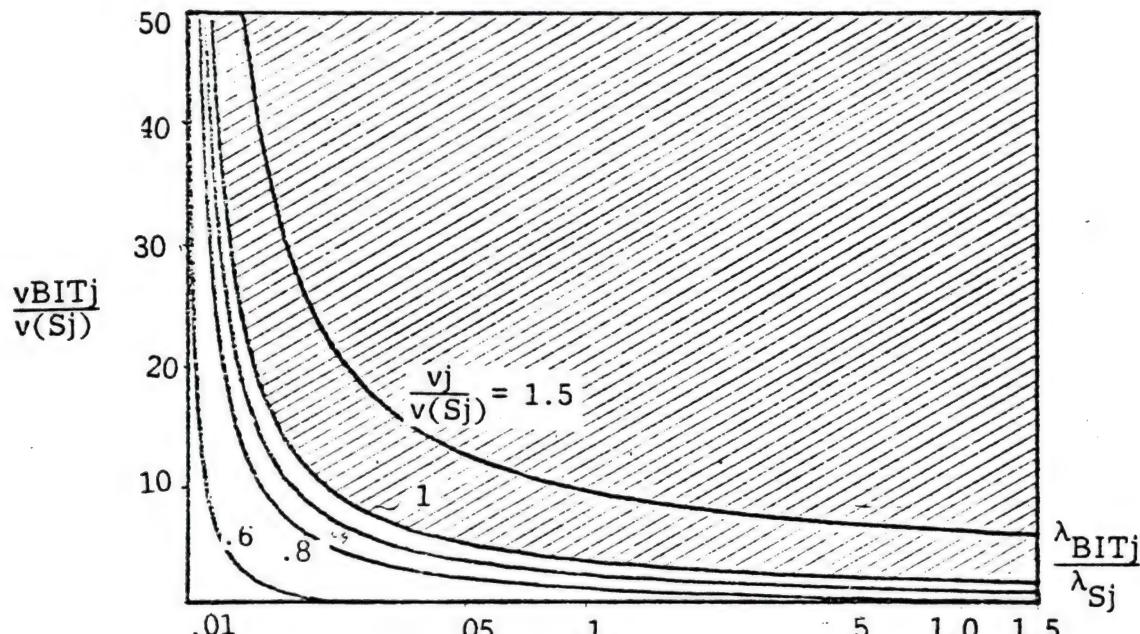


Figure 10: No false alarms.

Again the shaded area is the "no improvement" area. So even without false alarms the physical failures of the BIT can ruin our expectations for better and faster repairs. Note that two times better BIT cannot help if the mean time to repair the BIT is too long.

3. Ideal BIT which has negligible physical failure rate  $\frac{\lambda_{BITj}}{\lambda_{sj}} = 0$

$$y = .5 + (.2 + .7\eta)x \quad y = \frac{v_j}{v(sj)} \quad \eta = \frac{v(BITj)}{v(sj)} \quad (5.42)$$

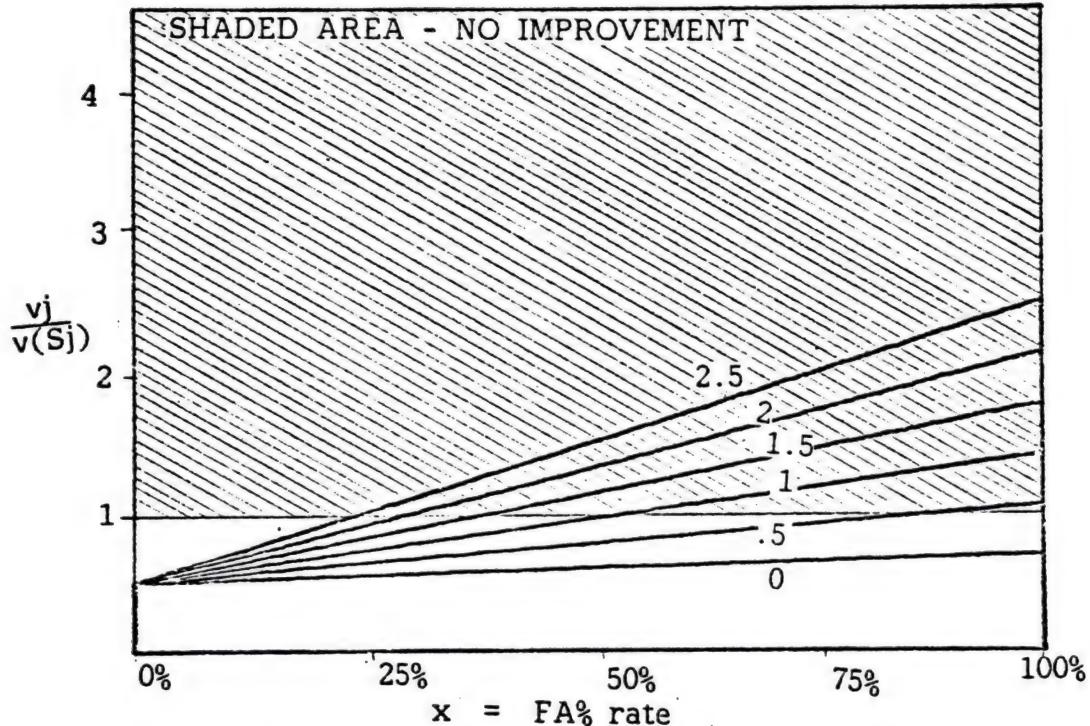


Figure 11: Ideal BIT with nearly no physical failures.

4. BIT is repaired very fast  $\frac{v(BITj)}{v(sj)} = 0$

$$B = (.5 + .7\xi) \frac{1}{1 + \xi} \quad y = B + [.7 - B]x \quad (5.43)$$

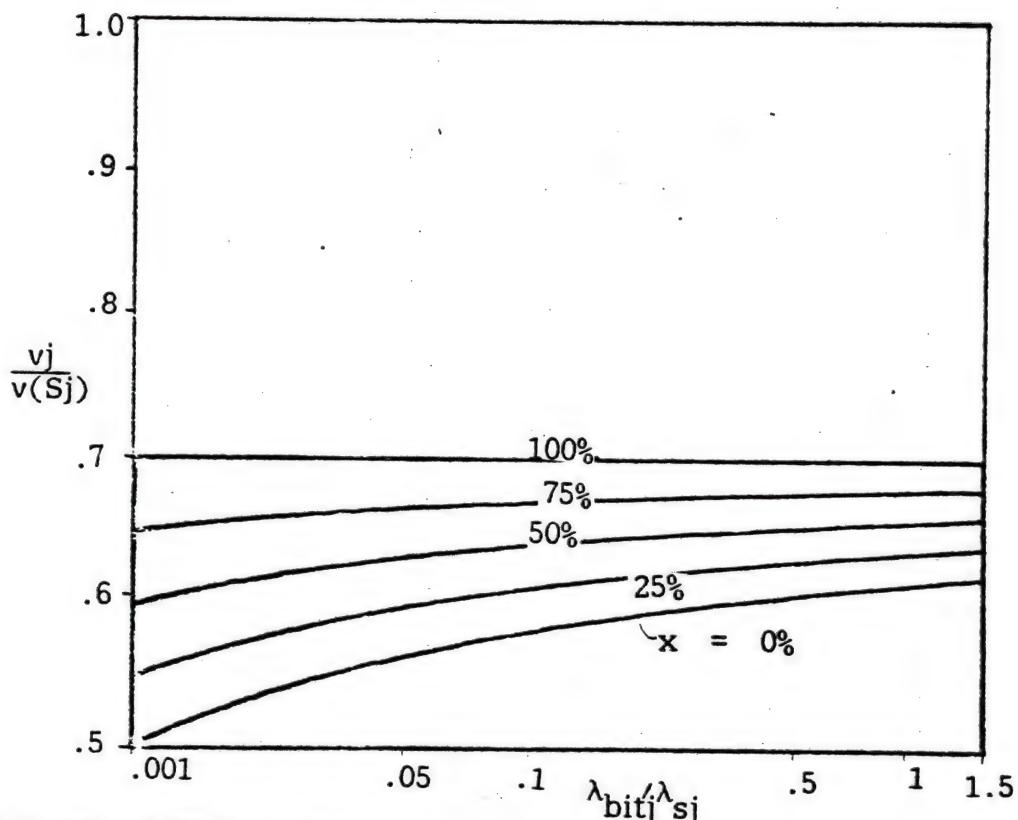


Figure 12: BIT fast repair.

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6. Marginal improvement plot  $y = \frac{v_j}{v(sj)} = 1$

To summarize all these plots, we plot the parameters with  $y = 1$  or  $v_j = v(sj)$  where no improvement in the repair time occurs.  $y < 1$  or  $v_j < v(sj)$  is all the area below particular curve in the plot.

Since  $v(\text{BITj})/v(\text{sj})$  and  $\lambda(\text{BITj})/\lambda(\text{sj})$  are the design parameters, this plot tells us roughly what rate of x-false alarms we can afford to tolerate for improvement of  $v_j$  over  $v(sj)$ . The plot provides the designer with some estimate of improvement. For example: Let say that at most 10% of FA can be tolerated; since  $\lambda_{sj}$  and  $v(sj)$  are usually given and thus  $A_{sj} = 1/(1 + v(sj)\lambda_{sj})$  is fixed. To improve the availability  $A_j$ , BIT should be added. If we can afford only two to ten times better parts in the BIT so that  $\lambda_{\text{BITj}}/\lambda_{\text{sj}}$  is .5 to .1 then in order to improve the  $A_j$ , we find from the figure 17 that MTTR for

BIT must be only 70% of MTTR of subsystem in order to improve anything at all. Usually parts for BIT will not be readily available and because BIT is higher quality and thus fails less, the technicians will spend more time to repair. If the time is bigger than MTTR the subsystem then is better to redesign the subsystem and omit the BIT. In the next chapter we discuss the availability change with the BIT.

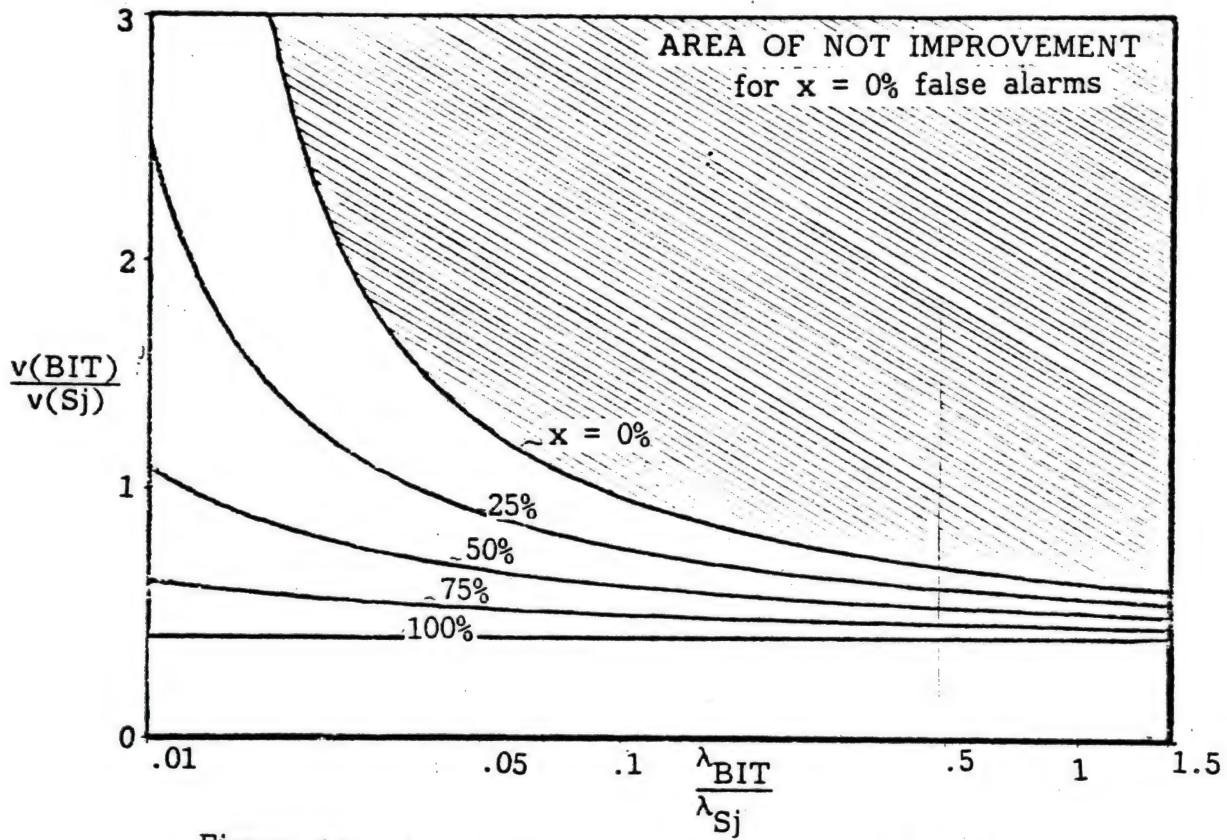


Figure 14: Area repair time improvement  $v_j < v(S_j)$ .

### V2.3 Estimates for the system availabilities

Since  $A_s = \frac{1}{1 + \frac{v_j}{\mu_j}}$  under assumption I and  $A_{av} = \frac{1}{1 + \sum_{j=1}^n \frac{v_j}{\mu_j}}$  we compute

system availabilities using the previously derived expressions for mean repair and failure of BIT-monitored blocks.

$$\frac{v_j}{\mu_j} = \frac{\mu(s_j)}{\mu_j} \cdot \frac{v(s_j)}{\mu(s_j)} \cdot \frac{v_j}{v(s_j)} = \frac{1}{\mu_j/\mu(s_j)} \cdot \frac{v_j}{v(s_j)} \cdot \frac{v(s_j)}{\mu(s_j)} \quad (5.44)$$

### Cases 2 and 3

In the cases 2 and 3 where false alarms do not influence the failure rate  $\mu_j = \mu(s_j)$ :

$$\frac{v_j}{\mu_j} = \frac{1}{\mu(s_j)/\mu(s_j)} \cdot \frac{v_j}{v(s_j)} \cdot \frac{v(s_j)}{\mu(s_j)} = \frac{v_j}{v(s_j)} \cdot \frac{v(s_j)}{\mu(s_j)} \quad (5.45)$$

$$\frac{v_j}{\mu_j} = [(.5 + \frac{\lambda_{BITj}}{\lambda_{sj}}) \frac{v(BITj)}{v(sj)} + \frac{\lambda_{BITj}}{\lambda_{sj}}] \frac{1-x}{1+\frac{\lambda_{BITj}}{\lambda_{sj}}} + (1+.7 \frac{v(BITj)}{v(sj)})x \cdot \frac{v(sj)}{\mu(sj)} \quad (5.46)$$

Under the assumption II of the suspended animation the above can be directly inserted into the expression for the average availability

$$A_{av} = \frac{1}{1 + \sum_{j=1}^n \frac{v_j}{\mu_j}}, \quad A_{av} = \frac{1}{1 + \sum_{j=1}^n \frac{v(sj)}{\mu(sj)}}$$

where  $A_{av}$  is the availability consisting only of the subsystems without the monitoring BIT. While under assumption II of independent block, we are left with some computations. We define the following

$$A_{BITj} = \frac{1}{1 + \frac{v(BITj)}{\mu(BITj)}} \quad (5.47)$$

$$A_{sj} = \frac{1}{1 + \frac{v(sj)}{\mu(sj)}} \quad (5.47)$$

$$A_j = \frac{1}{1 + \frac{v_j}{\mu_j}} \quad (5.47)$$

where  $A_{BITj}$  is the asymptotic value of the availability of the j'th BIT,  $A_{sj}$  is similarly the asymptotic value of the availability of the j'th subsystem or "previous" availability - availability from before the BIT was attached.  $A_j$  is the availability of the new - BIT equipped

subsystem - we called it block j.

$$\frac{v(\text{BITj})}{\mu(\text{BITj})} = \frac{1}{A_{\text{BITj}}} - 1 , \quad \frac{v(sj)}{\mu(sj)} = \frac{1}{A_{sj}} - 1 \text{ and } \frac{vj}{\mu_j} = \frac{1}{Aj} - 1 \quad (5.48)$$

we insert the above in the main equation and the result is:

$$\frac{\frac{1}{Aj} - 1}{\frac{1}{Asj} - 1} = \frac{x + \lambda_{\text{BITj}}/\lambda_{sj}}{1 + \lambda_{\text{BITj}}/\lambda_{sj}} + \frac{1 + .5(\lambda_{\text{BITj}}/\lambda_{sj})}{1 + \lambda_{\text{BITj}}/\lambda_{sj}}^{-1} (1-x) + .7 \frac{x}{\lambda_{\text{BITj}}/\lambda_{sj}} \frac{\frac{1}{A_{\text{BITj}}}-1}{\frac{1}{A_{sj}}-1} \quad (5.49)$$

Since we are really interested in a percentage of the availability improvement called  $\alpha$ , of the BIT equipped block versus the block without BIT, which is just the subsystem.

$$\alpha = \frac{Aj - Asj}{Asj} \quad (5.50)$$

let v be the expression in the main equation:

$$y = \frac{\frac{1}{Aj} - 1}{\frac{1}{Asj} - 1} = \frac{vj}{v(sj)} \quad (5.51)$$

then:

$$Asj = \frac{1 - (1+\alpha)y}{(1+\alpha)(1-y)} \quad (5.52)$$

Similarly, we define z as the ratio between the availability of the BIT and the corresponding subsystem availability:

$$z = \frac{A_{\text{BITj}}}{A_{sj}}$$

$$Asj = \frac{1/z - u}{1 - u} \quad (5.53)$$

where u is the expression in the main equation:

$$u = \frac{\frac{1}{A_{BITj}} - 1}{\frac{1}{A_{sj}} - 1} = \frac{v(BITj)}{v(sj)} \cdot \frac{\lambda_{BITj}}{\lambda_{sj}} \quad (5.54)$$

the above manipulations enable us to construct the nomogram, where

$\lambda_{BITj}/\lambda_{sj}$  is the parameter. To summarize:

$$v = \frac{v_j}{v(sj)}, \quad \alpha = \frac{A_j - A_{sj}}{A_{sj}} \quad (5.55)$$

$$A_{sj} = \frac{1 - (1+\alpha)v}{(1+\alpha)(1-v)}$$

$$v = \frac{x+\xi}{-1+\xi} \frac{1+5\frac{1}{\xi}}{1+\xi} (1-x) + .7 \frac{x}{\xi} u$$

$$\xi = \lambda_{BITj}/\lambda_{sj}$$

$$u = \frac{\frac{1}{A_{BITj}} - 1}{\frac{1}{A_{sj}} - 1} = \frac{v(BITj)}{v(sj)} \xi, \quad z = \frac{A_{BITj}}{A_{sj}}$$

$$A_{sj} = \frac{1}{\frac{z-u}{1-u}}$$

Obviously  $\alpha_{max}$  is achieved when  $A_j$  availability will be 1, from there we calculate maximum  $v$ , while maximum  $u$  is achieved when the BIT is 100% available. Usually we can estimate from experience  $A_{BITj}$  easier than  $v(BITj)$  - MTTR for BIT, but in any case the nomogram for determining  $A_{BITj}$  from  $A_{sj}$ , knowing  $v(BITj)/v(sj)$  and  $\lambda_{BITj}/\lambda_{sj}$  is added at the bottom. The plot might serve also for quick estimate of required MTTF and MTBF for BIT to achieve certain improvement.

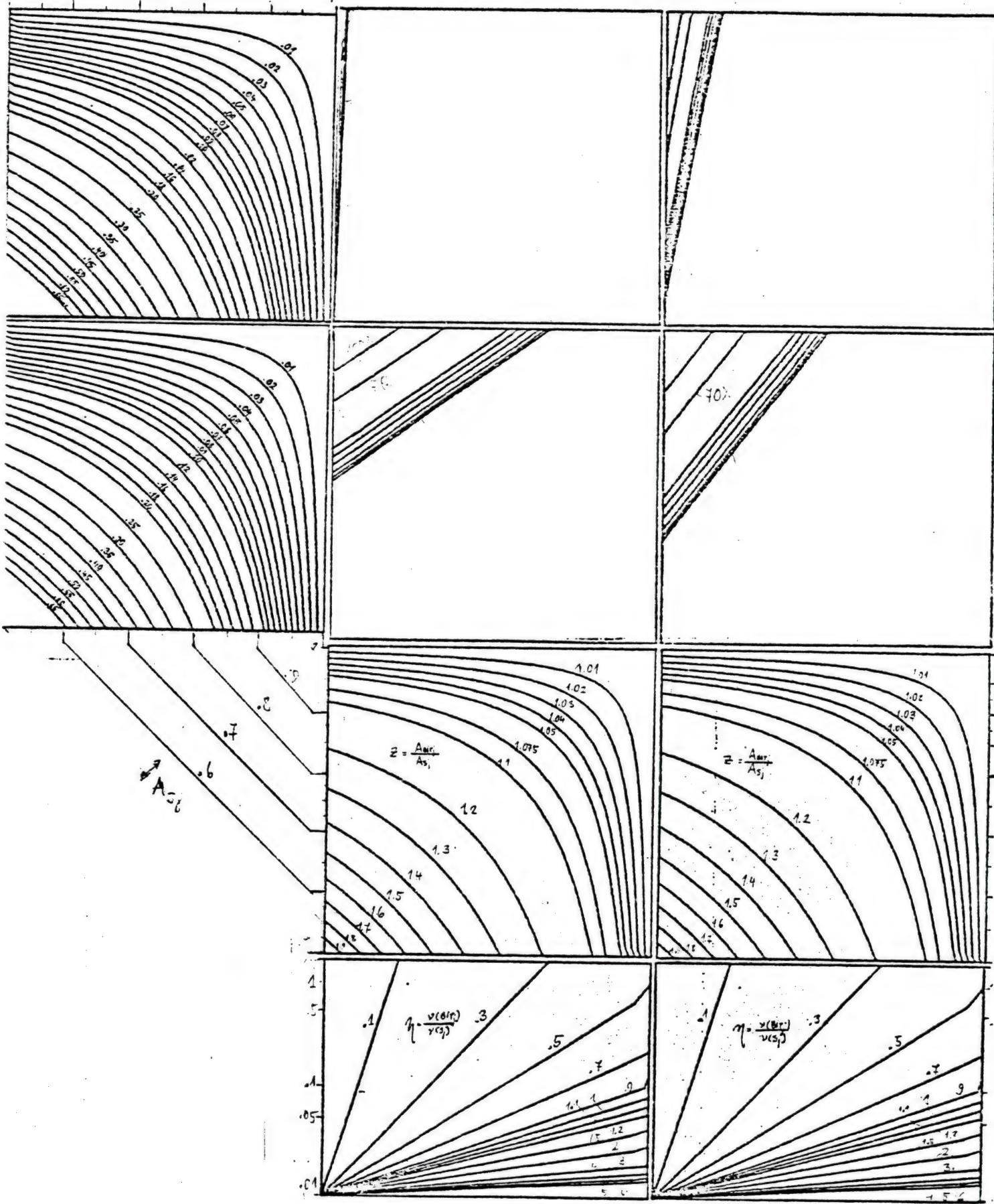


Figure 15: Case 2 and 3. Nomogram for % of improvement of  $A_j$ .

The above nomogram can be used to determine % of improvement from the design characteristics.  $\lambda_{BITj}/\lambda_{sj}$ , rate of false alarms x and both availabilities  $A_{sj}$ ,  $A_{BITj}$ . Also if we have in mind the desired percentage of reliability improvement of the availability, the needed values of the parameters can be obtained from the nomogram.

Since the symptotic value of the reliability importance  $I_j$  was defined (4.12)

$$I_j = h(1_j[A_i]) - h(0_j[A_i]) = \frac{\partial h([A_i])}{\partial A_j} \quad (5.56)$$

and we can asses easily overall improvement of the system availability

$$\begin{aligned} \Delta h([A_i]) &\equiv \sum_{j=1}^n \frac{\partial h(A_i)}{\partial A_j} \Delta A_j \\ \Delta h([A_i]) &\equiv \sum_{j=1}^n I_j \Delta A_j \end{aligned} \quad (5.57)$$

where  $\Delta h([A_i]) = h([A_i]) - h([A_{Si}])$  and  $\Delta A_j = A_j - A_{sj}$  where  $\Delta A_j$  can be obtained simply from the previous nomogram. From equation 5.52 we conclude that only the most important blocks with big  $I_j$  counts. So we concentrate only on improving these blocks, since others are not influencials ( $I_j = 0$ ).

The same is true under the assumption II of the suspended animation. Here we assumed the series connection, so that the worst subsystems are also the most important. We then concentrate to these blocks.

For the rough estimates of the marginal improvement, we specify  $\lambda_{BITj}/\lambda_{sj}$  and evaluate:

$$2 > \frac{1}{(1-B)A_{sj} + B} \quad (5.58)$$

$$B = \frac{1}{1 + .5 \frac{1}{\xi} + .7 \frac{x}{1-x} (1 + \frac{1}{\xi})} \quad \xi = \frac{\lambda_{BITj}}{\lambda_{sj}}$$

A figure 16 presents the obtained results. The plot is similar to the figure 14, only here the relation is between the availability of the subsystem  $A_{sj}$  and the normalized BIT availability  $A_{BITj}/A_{sj}$ . The nonfeasible area is clearly visible. For the fixed values of false alarm rates the area under the curve will yield improvement. This plot might thus serve as the quick orientation in the design.

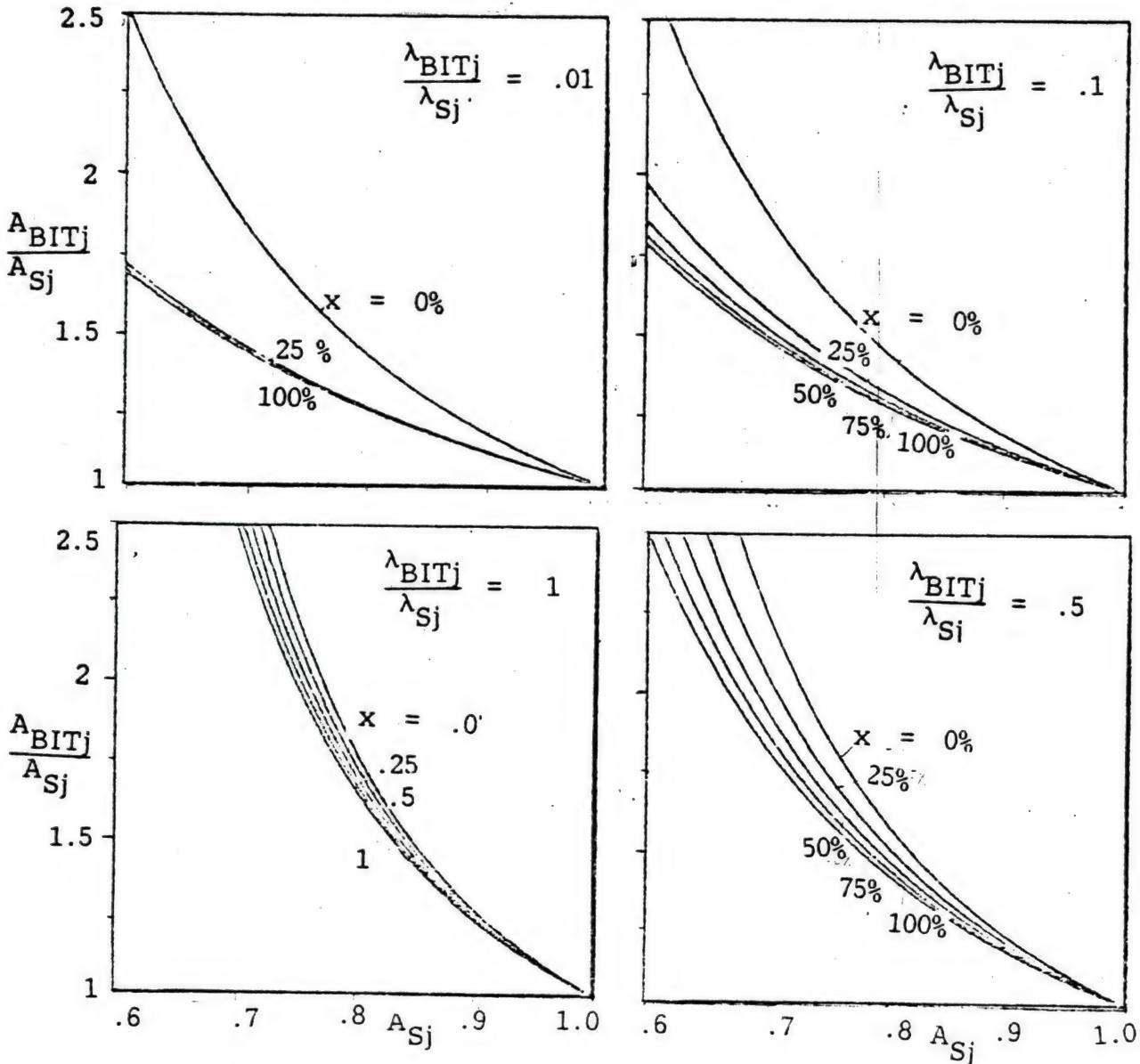


Figure 16: Plot of marginal improvement.

Since the percentage of the reliability improvement  $\alpha = \Delta A_j / A_{sj}$  for the j'th block is expressed through the set of equations it is also of some interest what is the influence or the sensitivity of  $\alpha$  to the parameters:

$$\alpha = \frac{A_j - A_{sj}}{A_{sj}}$$

$$\alpha = g(A_{sj}, \frac{\lambda_{BITj}}{\lambda_{sj}}, \frac{A_{BITj}}{A_{sj}}, x) \quad (5.59)$$

Now as before:

$$\Delta\alpha = \frac{\partial g}{\partial A_{sj}} \Delta A_{sj} + \frac{\partial g}{\partial \xi} \Delta \xi + \frac{\partial g}{\partial z} \Delta z + \frac{\partial g}{\partial x} \Delta x \quad (5.60)$$

where  $z = A_{BITj}/A_{sj}$ ,  $\xi = \lambda_{BITj}/\lambda_{sj}$ . And again we can plot these values.

We might be also interested in the percentage changes. Since the center of our discussion is the availability improvement or degradation, we cannot use  $\alpha$ -the percentage of improvement  $-\Delta\alpha/\alpha$  might blow up around zero. Instead we define:

$$\frac{\Delta A_j + A_j}{A_j} = \frac{A_{jnew}}{A_j} = \frac{\Delta A_j}{A_j} + 1 = 1 + \alpha \quad (5.61)$$

Since  $\Delta(1+\alpha) = \Delta\alpha$ , we determine sensitivity coefficients as:

$$\frac{\Delta\alpha}{1+\alpha} = \frac{\partial \ln g}{\partial \ln A_{sj}} \frac{\Delta A_{sj}}{A_{sj}} + \frac{\partial \ln g}{\partial \ln \xi} \frac{\Delta \xi}{\xi} + \frac{\partial \ln g}{\partial \ln z} \frac{\Delta z}{z} + \frac{\partial \ln g}{\partial \ln x} \frac{\Delta x}{x} \quad (5.62)$$

where  $\frac{\partial \ln g}{\partial \ln \xi} = \frac{\partial g/g}{\partial \xi/\xi} = \frac{\partial g}{\partial \xi} \frac{\xi}{g}$  and we can plot these weights as before.

Figure 17 presents the results. In the left column are the partial derivatives and in the right one are the sensitivity coefficients. Nearly perfect BIT's are on the top -  $\lambda_{BITj}/\lambda_{sj} = .01$  or

mean time to failure of a BIT is 100 times greater than that of the corresponding subsystem. On the bottom there is a plot for BIT's with equal quality - the mean time to failure for a BIT and its subsystem are equal. The graphs show that the contribution to improvement results from initial availability of subsystem  $A_{sj}$ , which is more or less given and it is the actual reason for the adding of BIT's. In the same category is also  $z = A_{BITj}/A_{sj}$  which measures relative quality of the BIT. Since failure rates are already expressed explicitly in the equations 5.55 and 57 the  $z$  also represents relation between mean time to repair for a BIT and its subsystem. As it is clear, the better the BIT's, better the availability of the block. False alarms still harm the availability but not of the order of other parameters. This is also true on the field, since "good" systems will mostly show false alarms and the personnel will get "used to" them, which is OK, when the failures are not catastrophic, or when they can be detected by some other way.

Similarly, the effect on the block availability of the BIT quality -  $g = \lambda_{BITj}/\lambda_{sj}$  decreases with the quality. In all the cases, the derivatives and the sensitivity coefficients increases drastically with false alarm rates. So by reducing false alarms we can expect better performance of the block. One remark is also necessary here. As maintenance personnel get used to false alarms, they simply don't trust the BIT indications any more. So false alarms might very well return our system to the previous non-BIT state, not to mention the loss of image of the producer (besides the loss of quality).

Figure 18, shows partial derivatives and sensitivity coefficients as functions of  $\xi = \lambda_{BITj}/\lambda_{sj}$  quality of BIT. There are no signifi-

cant differences between different  $\xi$  . The conclusion we can draw from the graphs is that good BIT quality influences improvement much more than bad ones.

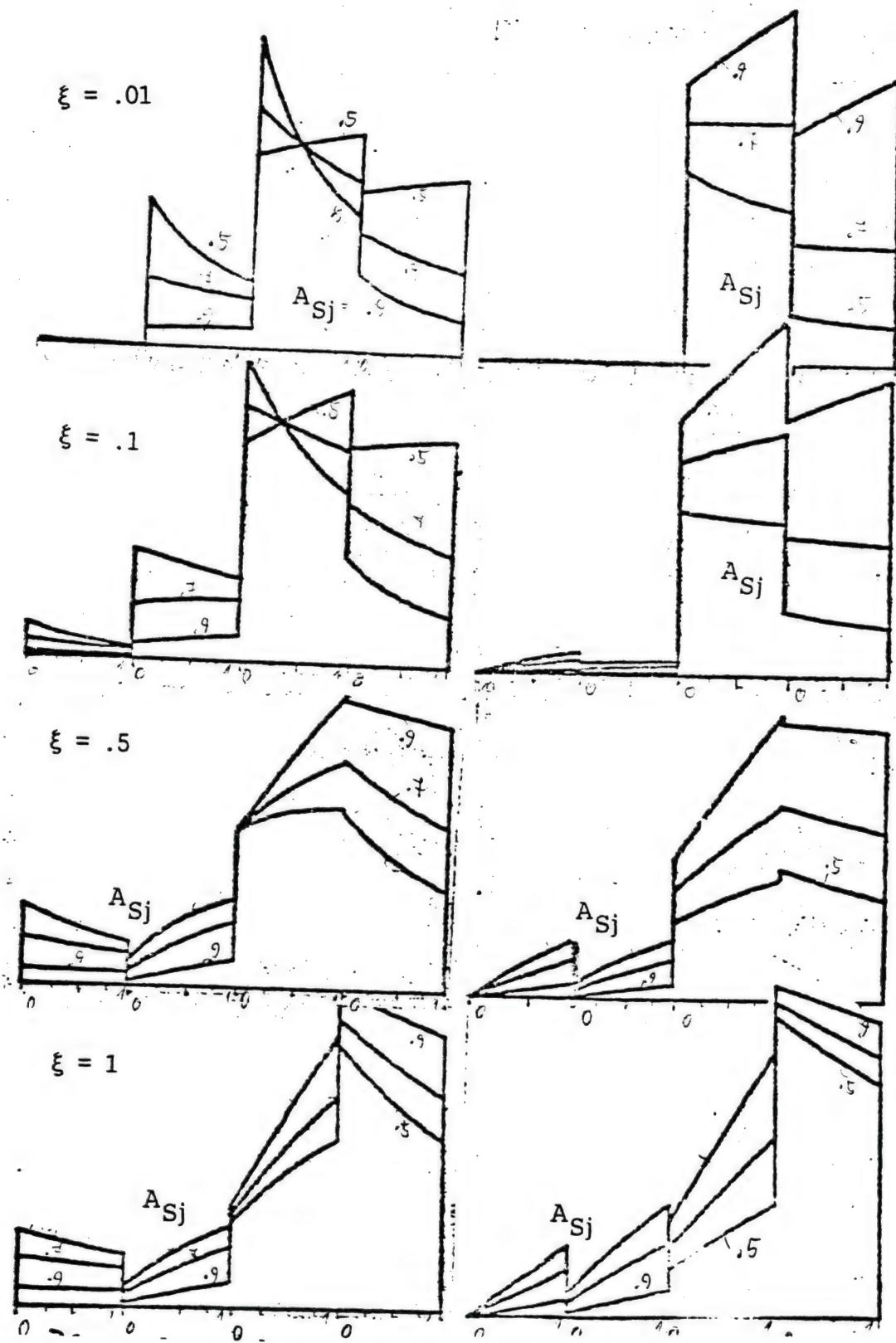
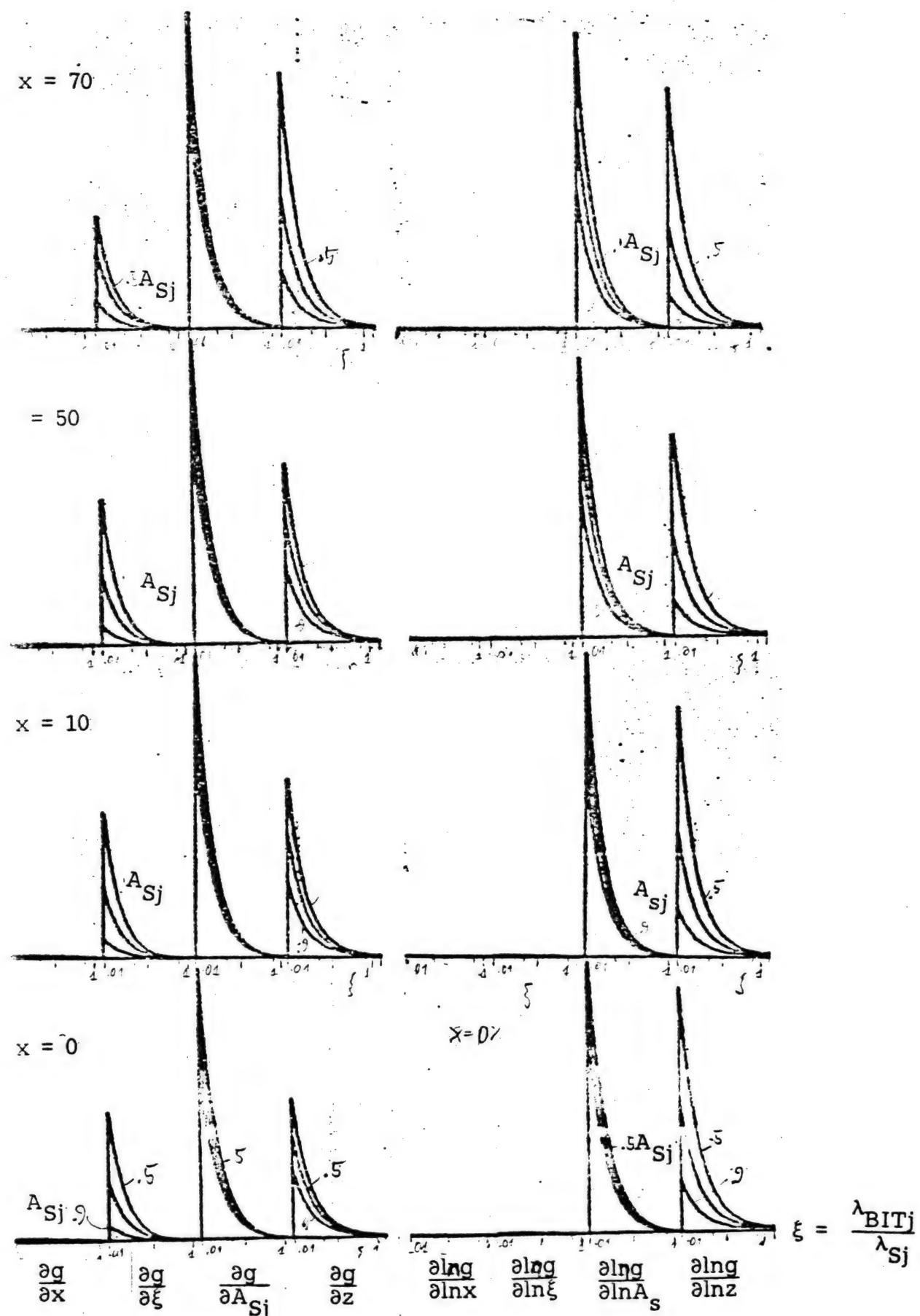


Figure 17: Partial derivatives of % improvement on left and sensitivity coefficients of ratio of improvement  $\Delta\alpha/1+\alpha$  on the right.



8: Partial derivatives and sensitivity coefficients as on the figure 1, but now as functions of  $\xi = \lambda_{BITj} / \lambda_{Sj}$ .

Combining the last results with the overall system, for example, we are able to roughly determine sensitivity of our system to let's say false alarm rate:

$$\Delta A = \sum_{j=1}^n I_j \frac{\partial g_j}{\partial x} \Delta x_j \quad (5.63)$$

$$\Delta A = \sum_{j=1}^n I_j \frac{\partial \ln g_j}{\partial \ln x_j} \frac{\Delta x_j}{x_j}$$

### Case 1

The only difference from Cases 2 and 3 above is the bigger failure rate of the j'th block since it is influenced by the false alarms.

As before:

$$\frac{v_j}{\mu_j} = \frac{1}{\mu_j/\mu(s_j)} \cdot \frac{v_j}{v(s_j)} \cdot \frac{v(s_j)}{\mu(s_j)}$$

Inserting equations 5.2 and 5.37 into above:

$$\begin{aligned} \frac{v_j}{\mu_j} &= \frac{1}{(1-x)/(1+\frac{\lambda_{BITj}}{\lambda_{sj}})} \left( 1.5 + \frac{\lambda_{BITj}}{\lambda_{sj}} \right) \frac{v(BITj)}{v(sj)} + .7 \frac{\lambda_{BITj}}{\lambda_{sj}} \frac{1-x}{1+\frac{\lambda_{BITj}}{\lambda_{sj}}} \\ &\quad + .7 \left( 1 + \frac{v(BITj)}{v(sj)} \right) \frac{v(sj)}{\mu(sj)} \end{aligned} \quad (5.64)$$

$$\begin{aligned} \frac{v_j}{\mu_j} &= .5 \left( 1 + \frac{\lambda_{BITj}}{\lambda_{sj}} \right) \frac{v(BITj)}{v(sj)} + .7 \frac{\lambda_{BITj}}{\lambda_{sj}} + .7 \left( 1 + \frac{v(BITj)}{v(sj)} \right) \left( 1 + \frac{\lambda_{BITj}}{v_{sj}} \right) \\ &\quad \frac{x}{1-x} \frac{v(sj)}{\mu(sj)} \end{aligned} \quad (5.65)$$

As before, the nomogram can be constructed using the above and  
(5.55)

$$y = \frac{v_j}{v(s_j)} \quad \alpha = \frac{A_j - A_{sj}}{A_{sj}} \quad (5.66)$$

$$y = \frac{1}{1-x} [ .7(x+\xi) + (1+.5 \frac{1}{\xi}) + .2x(\frac{1}{\xi} - 1.5)u ]$$

$$\xi = \lambda_{BITj}/\lambda_{sj}$$

$$u = \frac{\frac{1}{A_{BITj}} - 1}{\frac{1}{A_{sj}} - 1} = \frac{v(BITj)}{v(sj)} \xi, \quad z = \frac{A_{BITj}}{A_{sj}}$$

As before  $\alpha$  is the percentage of improvement (5.50)

$$\alpha = g(A_{sj}, \frac{\lambda_{BITj}}{\lambda_{sj}}, x, \frac{A_{BITj}}{A_{sj}})$$

And the plot of the partial derivatives and sensitivity coefficients is given on the next pages. Conclusions are similar than before, only the situation becomes much worse.

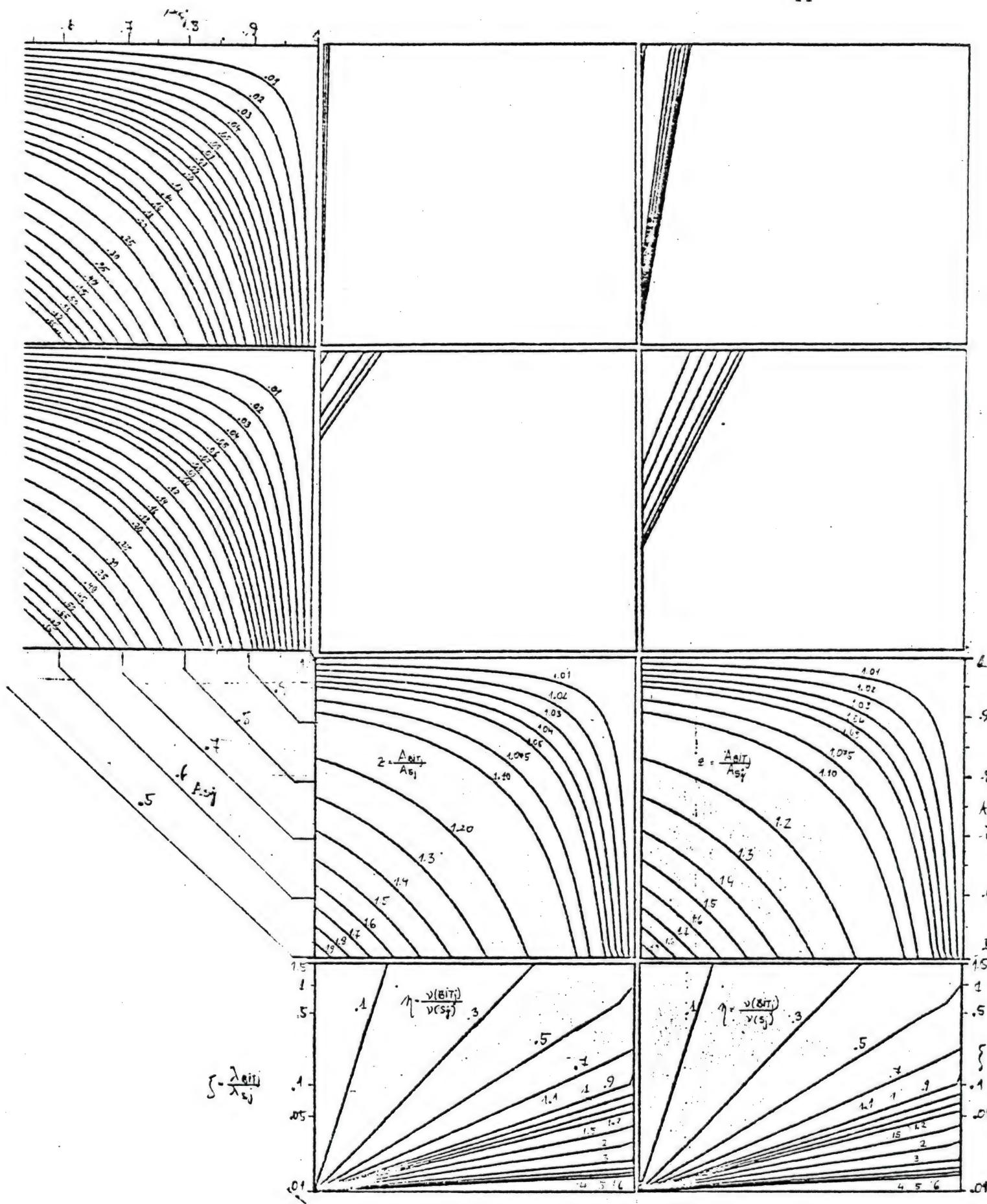


Figure 19: Same as 15, but for case 1.

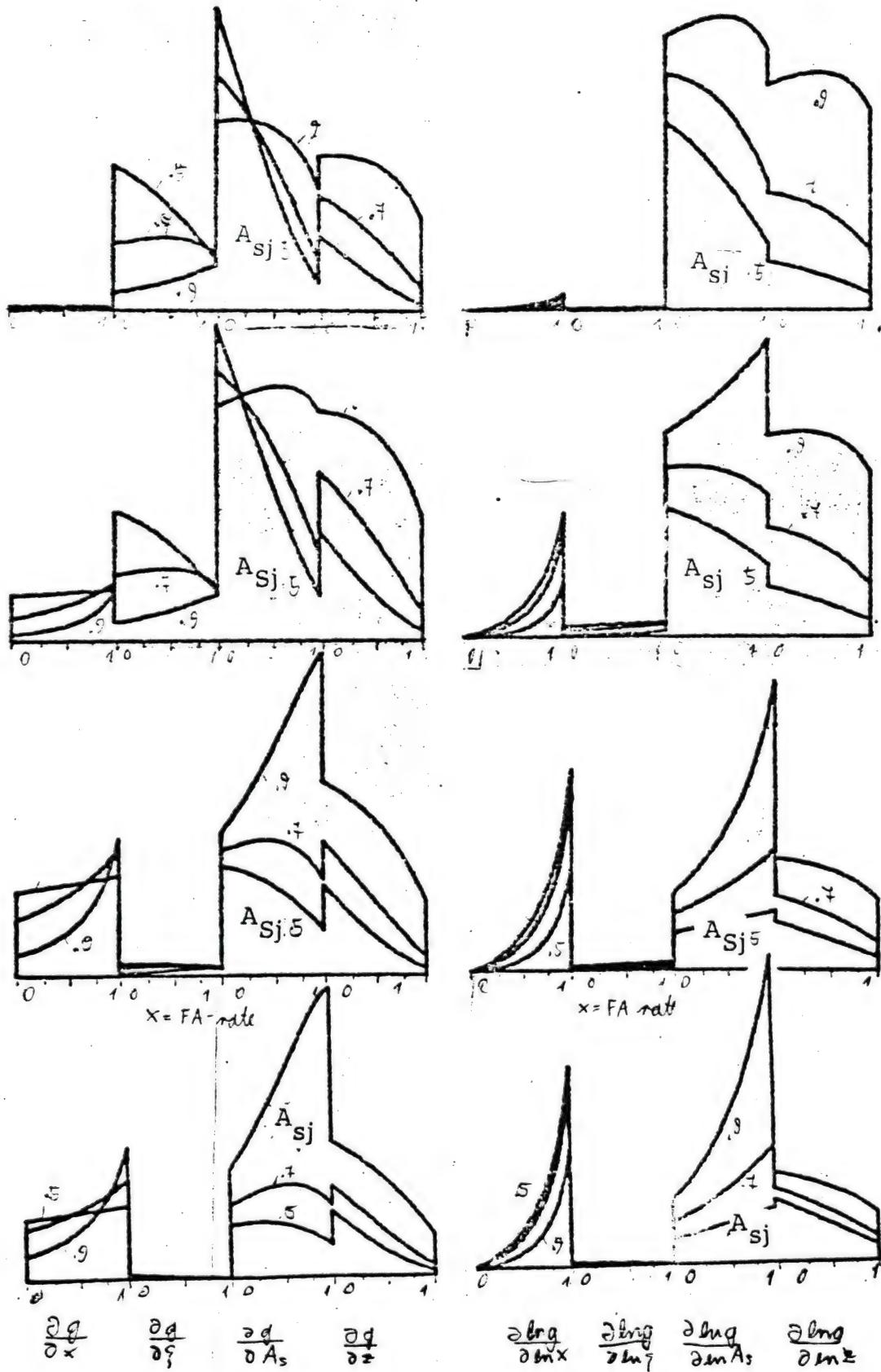


Figure 20: Same as 17, only for case 1.

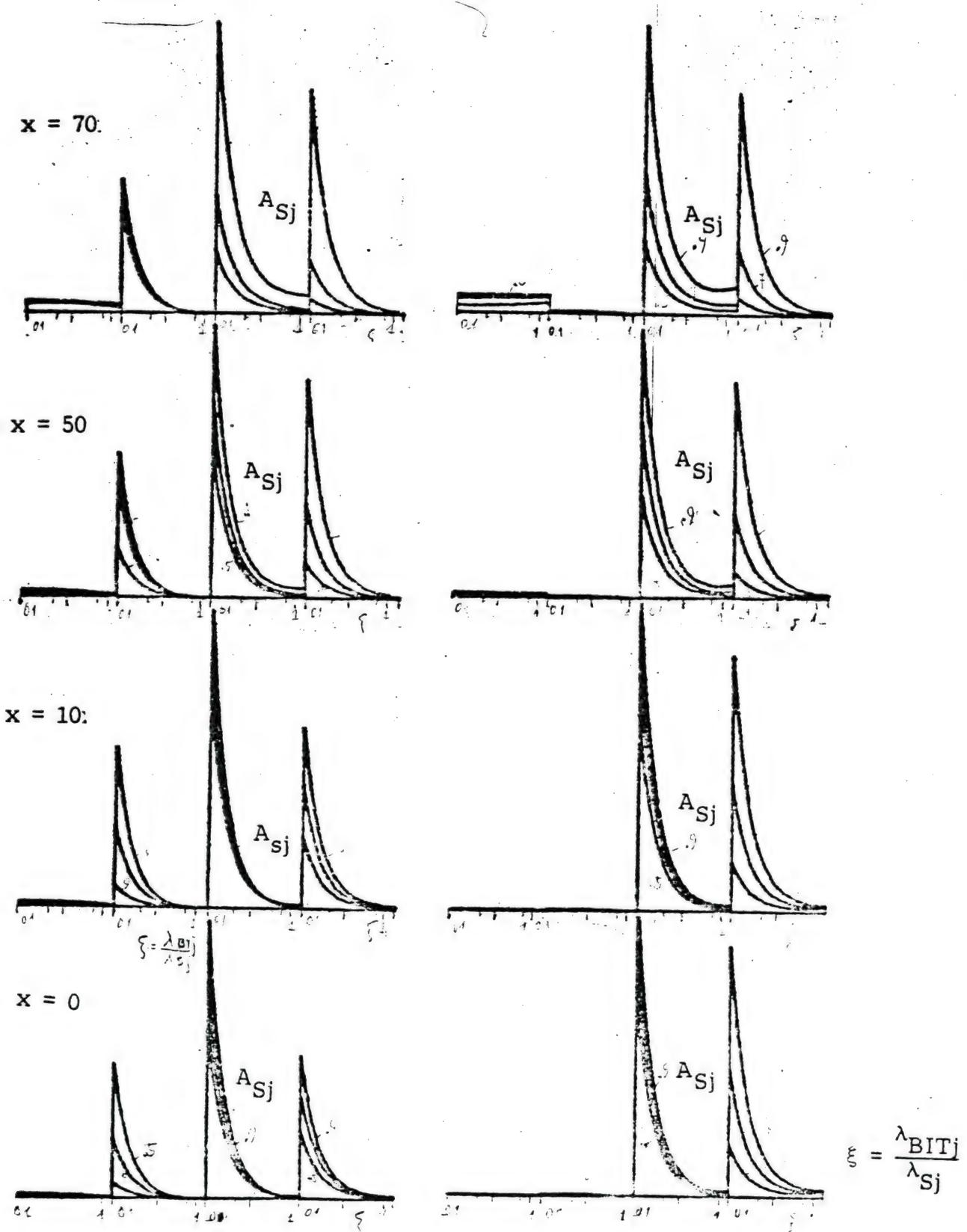


Figure 21: Same as 28 only for case 1.

Before we proceed with an example, we will show that one to one assumption of one subsystem monitored by one BIT is not very restrictive.

## VI STRUCTURES WHICH CAN BE REDUCED TO ONE SUBSYSTEM ONE BIT IN THE BLOCK

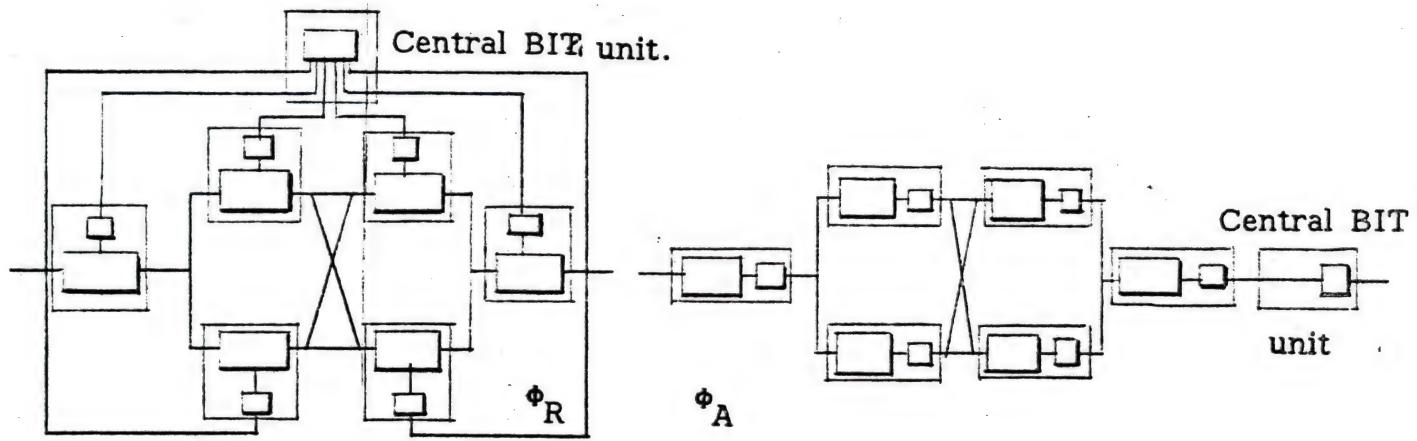
In this section we show that the previous discussion is much more general than it would seem from the restrictive assumptions. First we discuss a central BIT controller which communicates with each block's BIT. We also show how to deal with systems controlled by several sensors and where a single BIT monitors more than one subsystem.

### VI 1: A CENTRAL BIT CONTROLLER WHICH COMMUNICATES WITH EACH BLOCK'S BIT

Usually the design of a system with BIT capability consists of BIT's coupled closely to one or several blocks and a higher level central BIT system which will collect information, manipulate and store it. For instance, the airplane crew is interested in the status of their plane, without details which should be provided to maintenance personnel. The information displayed is therefore different for different users of the same system. To rule out unnecessary "repairs" because of the false alarms from the upper level, all BIT's in blocks are always provided with their own displays.

We can treat the above situation as before from the reliability and from the availability viewpoints. We can represent the central BIT unit as the BIT in d block where the subsystem is always available  $A_{SC}(t) \equiv 1$ , so that no repairs are needed.

$$\Phi_R([X_j(t)]^*) = \Phi_R([X_{sj}(t)X_{Bj}(t)]) \cdot (1 \cdot X_{Bi}(t)) = \Phi_R([X_{sj}(t) \cdot X_{Bj}(t)]) \quad (6.1)$$



**Figure 21:** Centralized BITj network. On left is reliability consideration where grey components are irrelevant. On the right there is availability viewpoint in which central unit can trigger maintenance actions.

where  $\Phi_R$  and  $[ ]^*$  are augmented function and vector. The rationale behind this is that whenever something is declared as "not OK" on the central screen, it is the technician's duty to verify what is wrong. So he is referred to the particular blocks.

The next section generalizes the discussion to similar treatment in the case where not every block is monitored for its operational status.

#### VI.2: BLOCKS NOT EQUIPPED BY BIT

The same subsystem, for some reason, is not covered by a BIT, we just introduce a fictitious BIT in the block. We have just to keep in mind that for such a block  $X_{Bj}^R(t) \equiv 1$  whenever the system is in the operation and that  $X_{Bj}^R(t) \equiv 0$  when the repair is concerned.

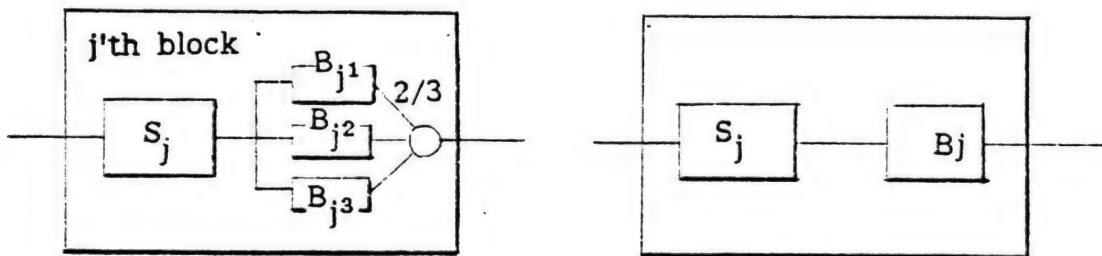
$$X_{B_j^R}(t) = \begin{cases} 1 & \text{functioning} \\ 0 & \text{maintenance} \end{cases} \quad (6.2)$$

Hence, the mean repair time is  $v(B_j^R) = 0$  since no time is really spent. All the other derivations stay the same.

In nearly every system we will encounter situations where some of the subsystems possess several BIT's to monitor them or when several subsystems are monitored by a single BIT. We elaborate on these situations in the next two sections.

### VI.3: SUBSYSTEM EQUIPPED WITH SEVERAL BIT'S

When complex subsystems are equipped with BIT's usually the BIT's consist of different sensors to monitor various operations of the subsystem. For example, the jet engine can be equipped with temperature, pressure, flow rates sensors which in addition to the regular measurements, will also provide information of what is wrong usually such arrangements are the majority voting or  $k$  out of  $n$  type. This kind of architecture is used to reduce the false alarms rate.



**Figure 23:** Typical situation where the  $j$ th subsystem is monitored by several BIT's. Here 2 out of 3. We can always reduce such situations to 1-1 or one subsystem, one BIT configuration.

Let  $\psi_{B_j}([X_{B_j}(t)])$  be the structure function of BIT's. Then we define the status of the composite BIT as:

$$X_{Bj}(t) \triangleq \psi_{Bj}([X_{Bj}(t)]) \quad (6.3)$$

where  $i = 1, 2, \dots, m$  is the number of BIT's in the  $j$ 'th block. Everything else stays the same.

A similar situation appears when several subsystems are monitored by one BIT.

#### VI.4: SEVERAL SUBSYSTEMS MONITORED BY THE SAME BIT

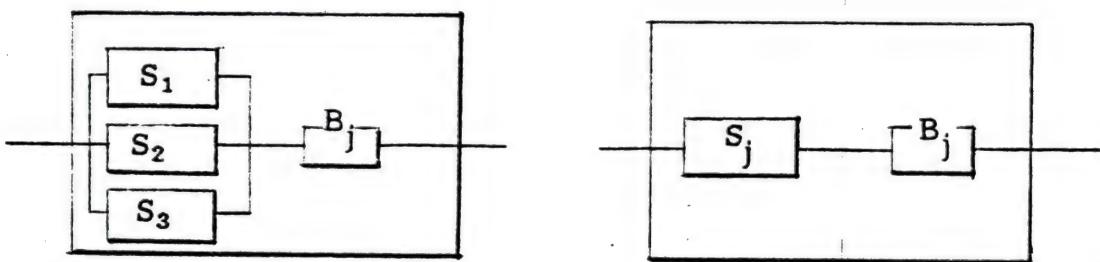
Good design practice will always try to avoid the use of a single BIT to monitor several subsystems, since when something is wrong, we have to find which one of the subsystems caused the problem. Usually such a case occurs when several identical subsystems are connected in parallel.

To treat the situation we build a  $j$ th block around each BIT and not around each subsystem.

As before  $\psi_{sj}([X_{sji}(t)])$  be the structure function in the  $j$ 'th block we define:

$$X_{sj}(t) \triangleq \psi_{sj}([X_{sji}(t)]) \quad i = 1, 2, \dots, m \quad (6.4)$$

where  $m$  is the number of the subsystems in the block.



**Figure 24:** Typical situation when several subsystems are monitored by one BIT. Here parallel structure of subsystems is assumed, which is the most usual situation. We always reduce the situation to 1-1 block arrangement.

Everything stays the same only the repair time must include terms due to the possible misclassifications.

Of course there might be other arrangements between BIT's and the subsystems, but the question arises about their effectiveness. If the BIT cannot be treated as one to one to the subsystems in the block, then the derived simplified analysis will not suffice.

To show the usage of one treatment developed here, we present the following example.

### VII. EXAMPLE

To illustrate the above model we discuss a highly simplified radar.

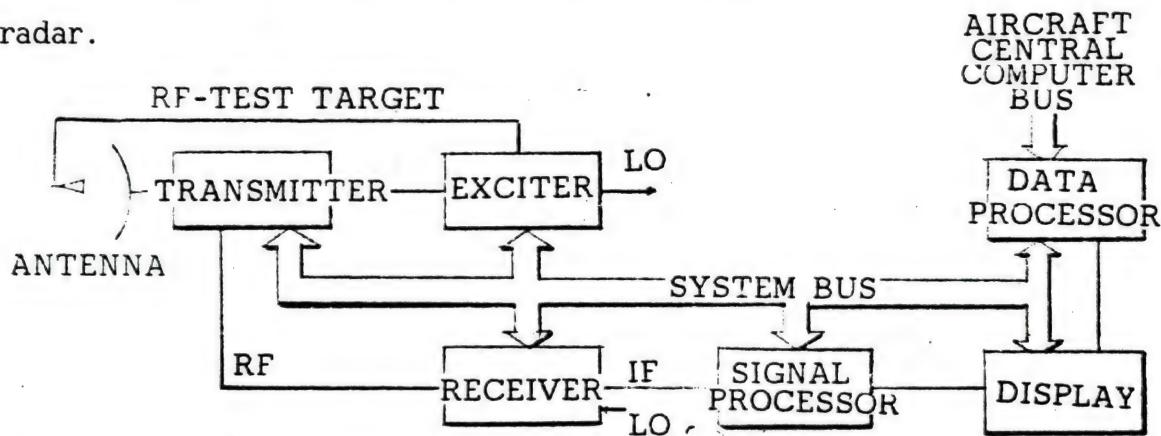


Figure 25: Functional sketch of the radar.

For proper operation, all the above subsystems must function properly, so the system has the series reliability structure:

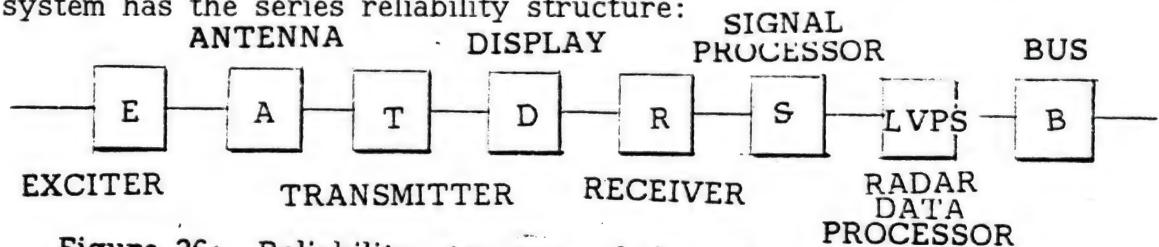


Figure 26: Reliability structure of the radar.

Let the (hypothetical) values of MTTF and MTTR of the subsystems be estimated as:

SUBSYSTEM	AVAILABILITY					IMPORTANCE
	MTTF	MTTR	$\lambda S_j v(S_j)$			
EXCITER	20	5	.25		.800	.444
ANTENNA	100	.5	.005		.995	.357
TRANSMITTER	50	2	.04	962		.369
DISPLAY	80	2	.025	976		.364
RECEIVER	11	5	.465	.687		.517
SIGNAL PROCESSOR	30	4	.133	.882		.402
DATA PROCESSOR	10	2	.20	.833		.426
BUS	80	5	.063	.941		.377

Figure 27: Data.

For a system with a serial reliability structure  $I_j = \frac{\partial h([A_i])}{\partial A_j} = \frac{\partial \prod A_i}{\partial A_j} = \prod_{i=j} A_i = \frac{A}{A_j}$ , so that the most important or influential are the worst components, which makes sense: the biggest increase we can expect when we change the most critical-the worst component

$$\Delta A = \sum_{i=1}^n I_j A_j$$

To incorporate the BIT in the Bus will not cause any additional effort. Some fixed pattern of 0's and 1's is sent through and checked. Similarly in the exciter we can provide a signal with known characteristics and its response can be observed on the screen. On the other hand, the central display does not need any BIT, since if it does not work that will be self-evident.

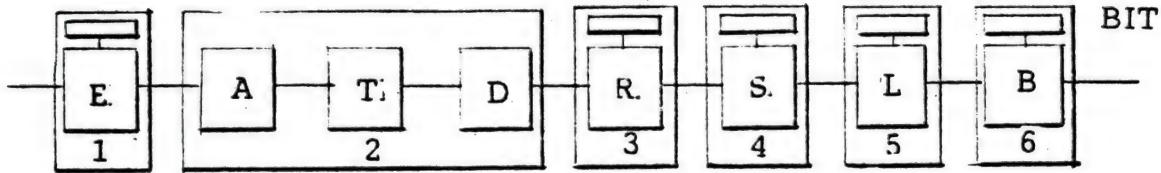


Figure 28: Organization in six blocks.

As soon as the decision is made about equipping blocks with BIT, we can draw a reliability diagram including BIT's. Note also that we can always withdraw BIT from consideration by using notation developed on page 36 in section VI.2.

We will consider four cases as described on the beginning (page 1) but we will reverse the order for convenience:

CASE 4: BIT indications ignored and no repairs are made on BIT. For example: the radar serves for detection of car speed.

CASE 3: BIT "not OK" indications must wait for the radar maintenance. For example: the radar on a patrol boat.

CASE 2: BIT checked up immediately and connected. For example: the radar on the carrier, technicians always available.

CASE 1: BIT indication "not OK" sends whole system out of operational readiness. For example: the radar in the guidance system of the missile; no officer wants to explode.

### SPEED CONTROL RADAR:

The case 4 is really not of interest; since there are no BIT repairs, nothing is really different from "classical" - no BIT systems.

### CASES 2 AND 3:

The difference between these two cases, is in the BIT failure rate. While in case 3, it is just the one which was estimated, in case 2 we have to count only the rate of BIT failures, which are not repeated by the time the system fails. We omit details here.

To get improvement in the availability, the MTTR's have to be reduced, since here the failure rate of the subsystem is not influence by the BIT's false alarms.

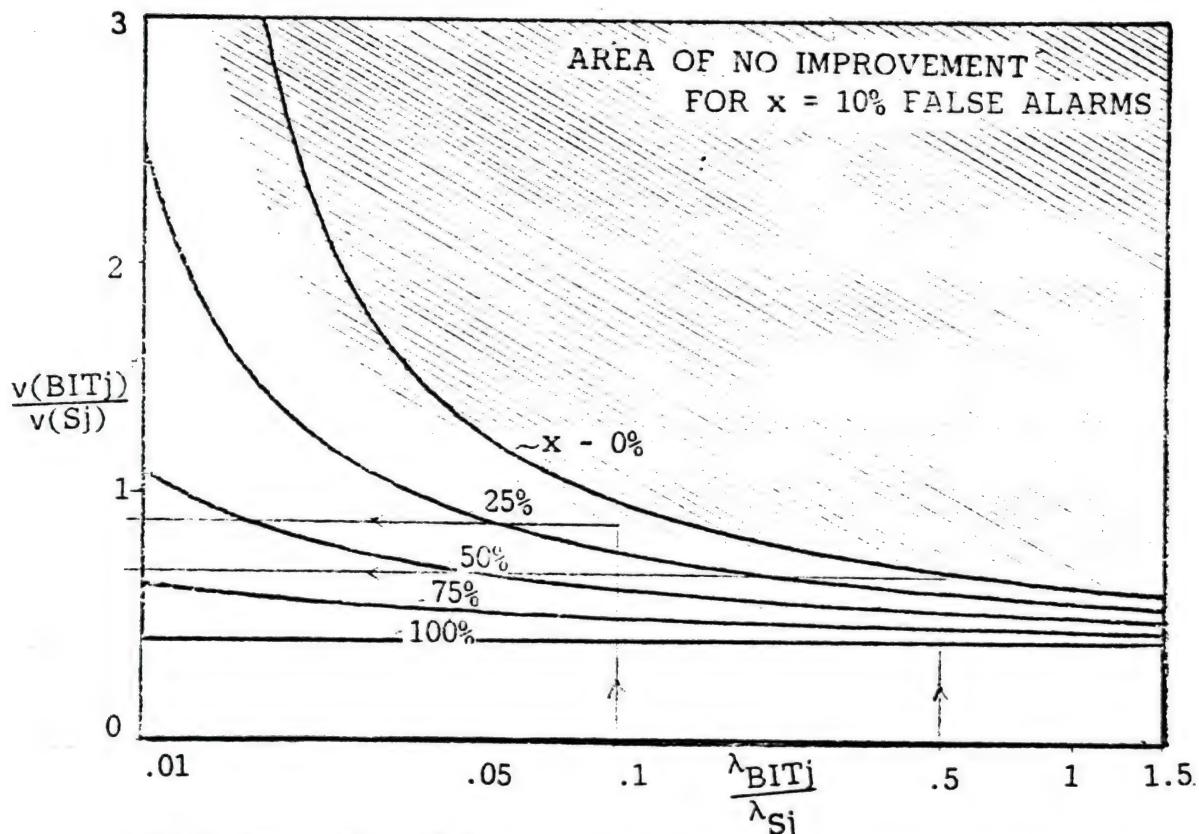


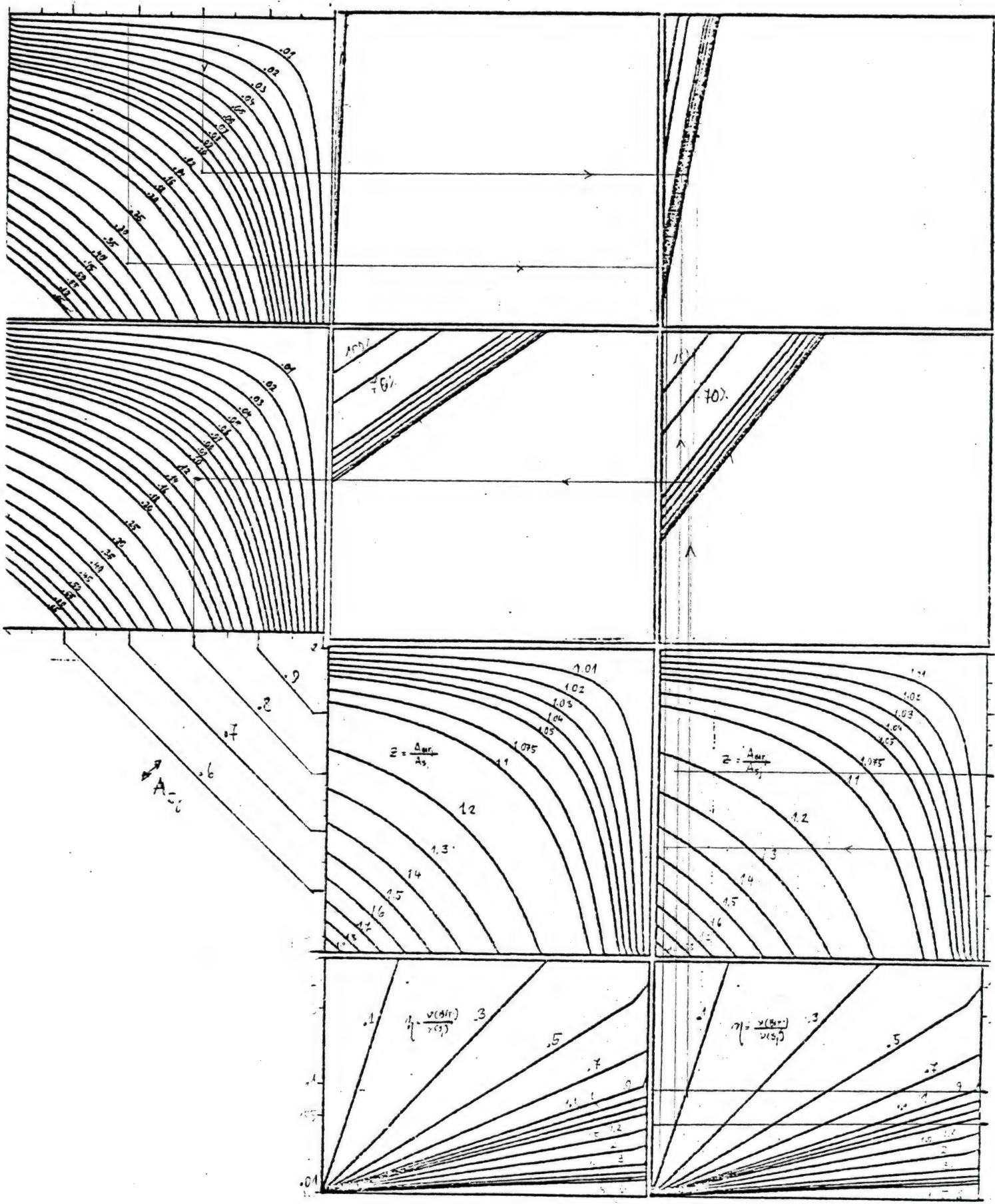
Figure 29: Area of improvement for cases 2 and 3 in the example: same as figure 14.

The shaded region in Figure 29 indicates the parameter correlations for this example for which BIT's give no improvement when the false alarm fraction is 10%. Since our BIT will be in general 2 to 10 times better than the original subsystem:  $\lambda_{BITj}/\lambda_{sj} = .5$  to  $.1$ , our BIT should have a MTTR less than .9 to .7 to get some improvement. Obviously this is not very easy. Spare parts and skill level of the personnel might every well prevent faster repairs. If this is the case, investing money in better subsystems might be more sensible.

We start the analysis with the worst subsystem. For quick orientation, we will use the nomogram in Figure 15. As soon as we get the percentage of improvement, we can evaluate its effect on the whole system by using equations (5.52).

$$\Delta A = I_j \Delta A_j$$

which is again just a quick orientation. Obviously we can also proceed the other way around. If we need a certain system availability, we can use  $I_j$  to approximately allocate the improvements. There are two things we have to keep in mind: first, this is just an approximation in terms of derivatives and so finally we have to repeat the detailed calculations. Second, estimated failure rates for mature systems are optimistic.



SUBSYSTEM					DESIRED BLOCK				ACTUAL BIT				ACTUAL BLOCK			
Block	Name	MTBF	MTTR	A <sub>dj</sub>	A <sub>dj</sub>	A <sub>av</sub>	A <sub>av</sub>	MTBF	MTTR	A <sub>bit</sub>	A <sub>av</sub>	A <sub>av</sub>	Sp. $\frac{A}{A_{av}}$	A <sub>av</sub> $\frac{1}{1+T_x}$		
1 E	EXCITER	20	.800	.444	.2	.90	.10	40	.952	1.17	7%	.15 + .17x	$\frac{1}{1.15 + 1.17x}$			
	A ANTENNA	100	.5													
2 T	TRANSMITTER	50	.2	.934	.359	6	934	/	/	/						
	D DISPLAY	50	.2													
3 R	RECEIVER	11	.5	.686	.514	1	90	.21 UNREACHABLE	20							
	S SIGNAL PROCESSOR	30	.4	.382	.402	4	90	.02	31%	.952	1.39	20%	22 + 3x	$\frac{1}{1.22 + 1.39x}$		
5 L	DADAR DATA PROCESSOR	10	.2	.923	.426	3	90	.07	8%	.968	1.10	9%	12 + 0.35x	$\frac{1}{1.12 + 1.10x}$		
6 B	BUS	80	.5	.941	.377	5	90	/	/	.909	1.09	1%	.5 + .36x	$\frac{1}{0.5 + 0.36x}$		
$A = \prod A_{d_j} = .535$					A = .72				A = .349				-10% FA rate			
$A_{av} = \frac{1}{1+T_x} = .461$																

Figure 31: Data - case 2, 3.

In the figure 30 we sketched how we got the required numbers for the desired reliability. Note it is not always possible, even with 100% BIT availability to get desired blocks availability. The reason is in our assumption that approximately 50% of repair time will go to the failure detection and isolation in the long run. Also the values are only estimates because of paper imprecision and since false alarms were just roughly taken as 10%.

When the actual BIT data are obtained (as before we introduce arbitrary data just for the example), we can look back for the % of the block availability improvement. We see much less improvement than desired.

The last column in Fig. 31 represents actual availabilities of the block, but x is left as the variable. Obviously every block will

exhibit a different percentage false alarm rate, but for the illustration on the influence of false alarms on the whole system availability, we evaluate:

$$A = \prod_{j=1}^n A_j = \frac{1}{.934(1.116+.17x)(1.22+.3x)(1.12+.034x)(1.5+.35x)(.078+.03x)}$$

$$A_{av} = \frac{\frac{1}{n}}{\frac{1}{1+\sum \xi_i}} = \frac{1}{2.141+.883x}$$

Figure 52 represents the above equations. The average availability in the assumption II of suspended animation is always greater than the availability under the independence assumption. Since  $\xi_j = v_j/\mu_j$  are positive:

$$A = \prod_{j=1}^n A_j = \frac{1}{\prod_{i=1}^n (1+\xi_j)} = \frac{1}{\sum_{j=0}^n (n) \xi_j^j} = \frac{1}{1+\sum \xi_j + n \xi_j^2 + \dots} < \frac{1}{1 + \sum_{j=1}^n \xi_j} = A_{av}$$

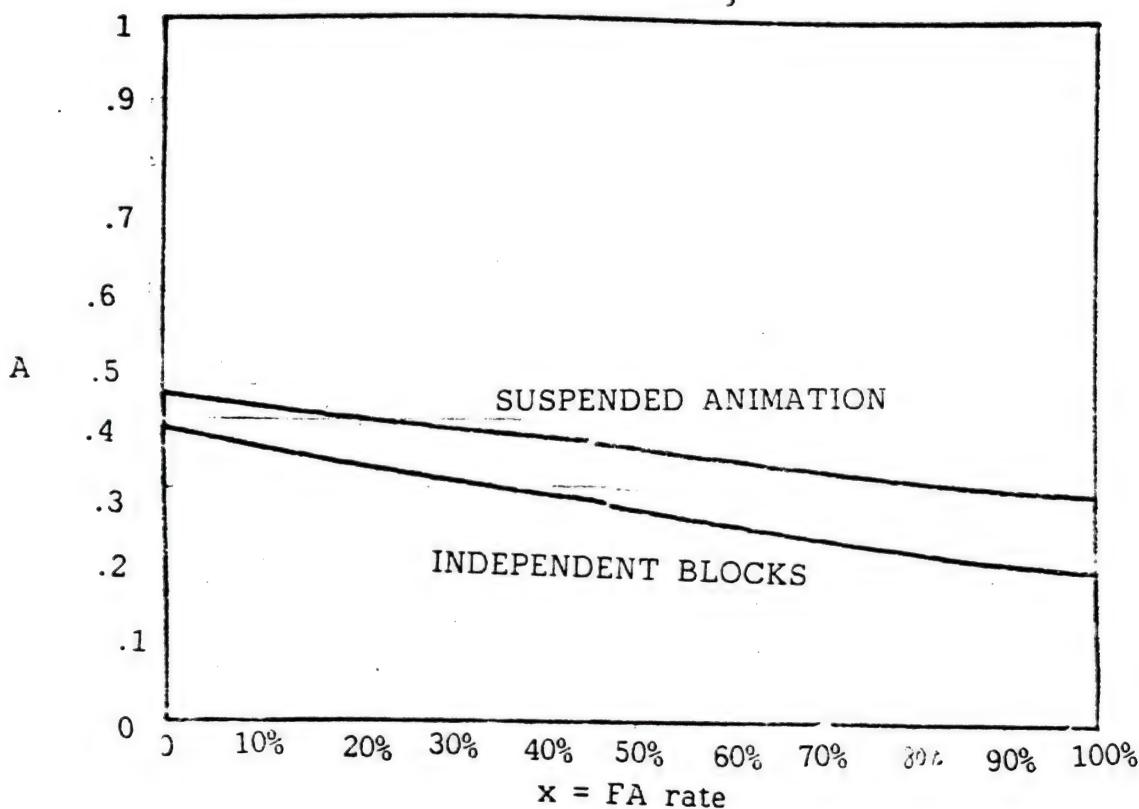


Figure 32: System Availability as a function of common FA-rate.

Figure 32 shows very pessimistic results, as a consequence of the data used in the examples. We see first that the availability with BIT might be worse as we discussed before. Second and more important is the result that, the personnel will not bother with large false alarm rates, since the false alarms just confuse them, and by ignoring the BIT's and repairing only the subsystems they will actually increase the availability of the system. Note also that this example was picked this way and other cases might be more optimistic.

#### Case 1

Here the false alarms influence also the failure rates and we repeat the above analysis step by step:

$$A = A_j = \frac{1}{.934(1.19+.37x^*)(1.27+.59x^*)(1.14+.23x^*)(1.27+.42x^*)(1.14+18x^*)}$$

$$A_{av} = \frac{1}{1+\sum \xi_i} = \frac{1}{2.08+1.79x^*}$$

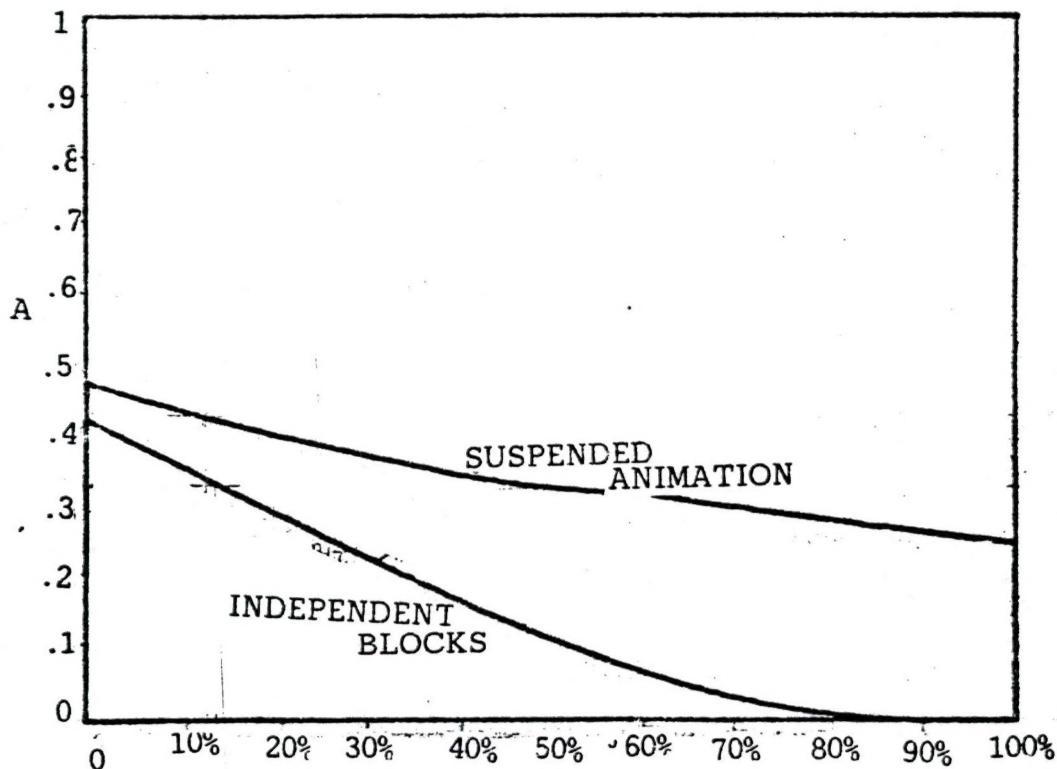
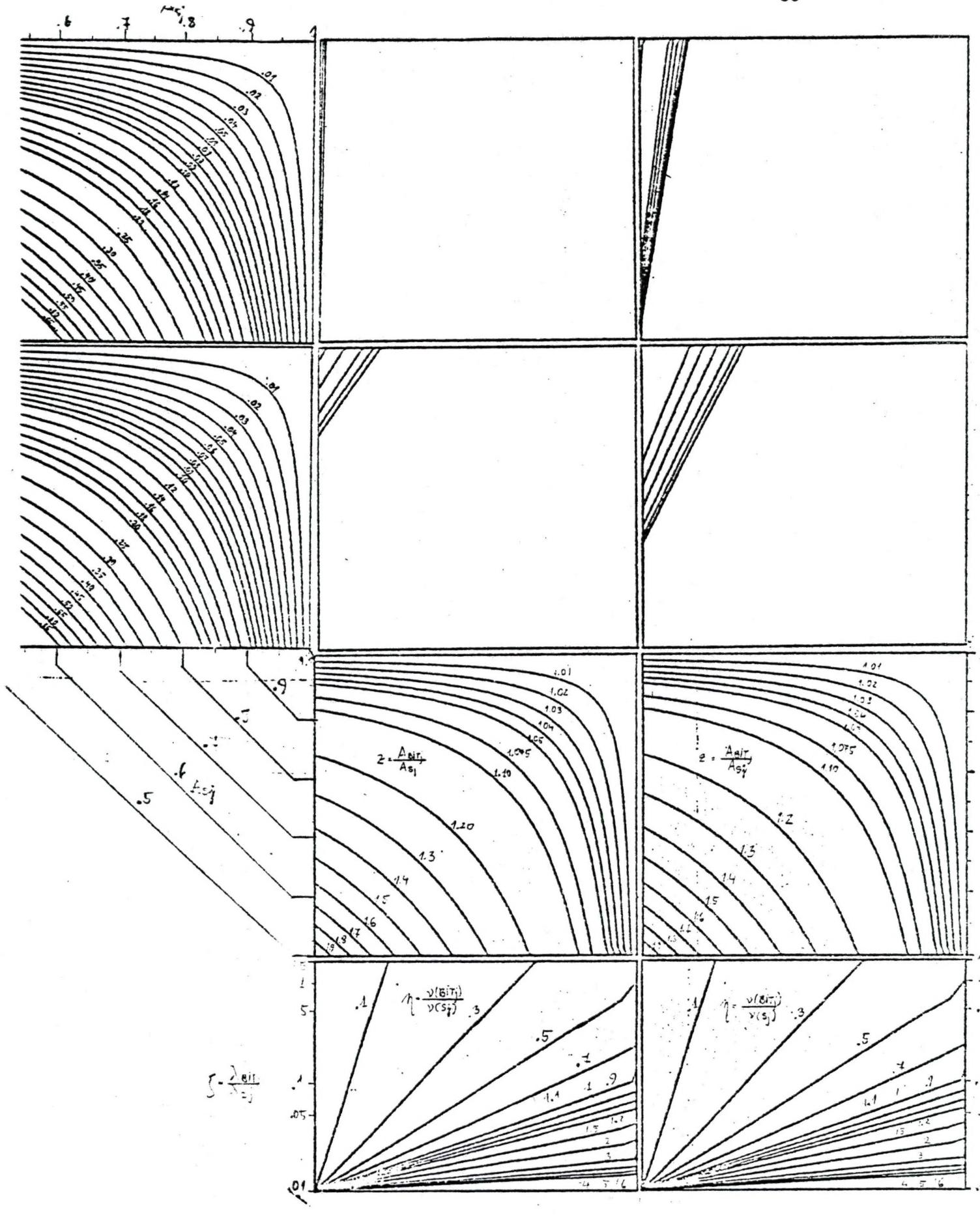


Figure 34: System availability, as 32, only case 1.



## VIII CONCLUSIONS

In the development of the model the following assumptions were made to simplify the derivation and to enable use of the available data bases:

- System configurations can be reduced to one BIT- one subsystem blocks.
- A failed BIT has the same effect on the block status as the wrong BIT indications, when the system is maintained.
- Both the subsystem and BIT are complex systems themselves, so that approximately constant failure rates might be used.

There was no assumption on the distribution of repair times. Although the division of MTTR among tasks was used, the same method can be used with different particular applications.

The resulting model is:

- simple
- easy to use and easy to understand
- computable: Data for evaluation are standard, so all existing data bases can be used, while the false alarm rate is treated as a variable, enabling all the conclusions from other studies to be used.
- consistent: It predicts the situations encountered in practice, where ignoring BIT might speed up repairs and so increase availability.

The examples presented show that built in tests should be introduced with care and far from everywhere, and all the time. The system availability is influenced most by the subsystem availability without BIT, so if the false alarms cannot be kept at minimum, or if BIT's are not much better than subsystems, it will be much more effective to invest in basic subsystems availability rather than to use BIT's. Also, it will be the most productive, to install BIT where digital circuitry is already available so that costs will be minimal. Since false alarms seriously degrade system availability, the failure isolation property is more useful than the failure detection. The BIT's are coming.